SCIENCE, SETI, AND MATHEMATICS

The material on astrobiology and SETI is accurate, clear and comprehensive, and up to date, stressing points that will delight and inform the uninitiated."

Albert Harrison, University of California

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Preface

This book is intended for my colleagues in the humanistic and natural sciences who share my interest in the search for extraterrestrial intelligence (SETI). It is about the role mathematics might play in this endeavor. Since I am writing for a wide audience, an audience of people with very diverse backgrounds, I have focused on ideas and avoided mathematical symbolism and technical jargon. No prior knowledge of mathematics is assumed and, since this subject may be new to many of my readers, I also present the history of, and the science behind, this search. My goal is to stimulate a discussion, among scientists interested in this area, of the ideas presented here.

Many contend that a great deal of our mathematics would be understandable, even familiar, to the members of any technologically sophisticated race—the only kind of society our current methods of searching will enable us to find. I examine this contention in detail. The astronomical environment of our planet, in particular our large moon, human evolutionary history, and our reliance on the sense of sight, have all influenced our mathematics. The subject is very much a part of our humanity, somewhat like our music and art. But mathematics has a way of becoming useful either as a model for some aspect of reality or in solving practical problems, and it can be more easily communicated to another, distant, society. I have tried to show that, in doing so, we say quite a lot about ourselves.

The early workers in SETI were concerned with the technical problems of sending and receiving radio signals across inter-stellar distances. Slowly, however, the deeper questions inherent in this endeavor rose to prominence: questions about the possible nature of extraterrestrial intelligence, the nature of language, and the philosophical/psychological motivation for this search.

In recent years these questions have attracted scholars from a remarkably wide variety of disciplines. Several recent books[1] contain articles written by philosophers, psychologists, anthropologists, archaeologists, artists, and religious scholars. These scholars bring valuable insight into the many deep problems posed by SETI. As we broaden the scope of our discussions, however, it is important to remember the realities of this endeavor. Our method of searching, the radio telescope, restricts the kind of society we might contact to those capable of sending electro-magnetic signals over inter-stellar distances (yes, some search for optical signals, others for evidence of alien technology, but communication, if it occurs, will be by some form of electro-magnetic radiation). Thus the early insights of astronomers, physicists, and mathematicians are still relevant and provide a framework for ongoing research. In this book I try to bring the early work to the attention of those new to the field. Also, at the end of each chapter I have a section labeled “remark.” Here I present some aspect of mathematics that, I think, might illuminate the ongoing discussion.

Anyone who expresses an interest in SETI is, sooner or later, confronted by someone, sometimes a very belligerent someone, who claims the subject is inane and pointless. As “proof” such people will relate stories of UFO (unidentified flying object) sightings that, they claim, show that aliens exist and visit us often. This can be very disconcerting, especially if it happens when one is giving a public lecture. But some familiarity with the major incidents shows such people and anyone else listening that you are neither ignorant of, nor afraid to face, these “facts”—just not impressed by them.

Unfortunately, in the minds of many, SETI and UFOs are related. This is not so, and I think the best way to demonstrate this is to present some of the evidence for UFO visitation; this evidence is essentially just a collection of stories. The reader is invited to reach his or her own conclusions as to whether or not these stories are evidence of extra-terrestrial visitation. Personally I am a skeptic. More precisely, I don't believe that those who say UFOs are alien spaceships have proven their case. The reasons for my skepticism are presented throughout the book, most explicitly in Chapter 9.

At this time I would like to thank Dr. Harry Lataw, Jr. who read an early version of this book and made many helpful suggestions. I owe a great debt to Dr. Al Harrison who went over the manuscript chapter by chapter and gave me many insightful comments, and to Christina Carbone, of the computer support staff at the University of Arizona, who was always helpful in answering my technical questions. Finally, I must thank my wife Marilyn for her encouragement and patience during this rather lengthy and often arduous project.

Note

1. *Archaeology, Anthropology, and Interstellar Communication* will appear in the NASA history series. *Between Worlds,* which will be published by M.I.T. press. *Communication with Extraterrestrial Intelligence* was published by SUNY Press, and *Civilizations Beyond Earth* was published by Berghahn Books.

Chapter 1

Where Are We?

This is a book about humanity's responses to the "Great Silence"—the fact that no sign of intelligent life beyond earth has yet been found. The most obvious of these is the scientific search for extraterrestrial intelligence (SETI). This search, as those involved in it are quick to point out, has nothing to do with unidentified flying objects (UFOs), or crop circles, or stories of weird little creatures intent on examining the genitalia of every human they come across. Certain incidents, however, are invariably asked about whenever SETI is discussed. We examine these incidents in several of the ensuing chapters. Failing to do so is like entering a room and trying to ignore the elephant in the corner.

The universe as revealed to us by modern science, the answer to the question "Where are we?" is disheartening. It is beautiful, fascinating, and endlessly surprising, but it is cold—cold, indifferent, and achingly lonely. We who were once—so we thought—the apex, the goal of all creation, find that we are the denizens of an ordinary world, circling a typical star near the edge of a galaxy—one of an estimated hundred billion galaxies in the observable universe. Our local environment, our solar system, is an intricate structure consisting of planets, moons, asteroids, and comets, but, apart from the Earth, it appears to be lifeless. Is intelligent life just an exceedingly rare accident? Are we the only ones here to appreciate the grandeur of creation? Is there no element of warmth, of compassion somewhere among the four hundred billion stars that make up the Milky Way galaxy? Perhaps not, but this is too terrible a truth to accept without a fight; so some of us will ceaselessly search the skies, looking for an intelligent signal, undeterred by the possibility that we may never find one.

But what could we possibly share with an alien race? Two things come to mind: Our mathematics, and basic physical science. Since we and any alien race certainly share the same physical universe, and any race we contact must have something like the radio telescope in order to respond, it seems reasonable that we share some science. But basing a language on this requires that we figure out how to communicate the basic human units of measurement. It doesn't help to tell someone the Sun emits so many calories per hour unless he or she knows what an hour is and what a calorie is. As for mathematics, we would expect that any society with the ability to send radio waves over inter-stellar distance would know how to count. The numbers we count with, and their properties, can be used to develop a simple language. This language, together with some facts from chemistry and physics, may enable us to communicate the basic human units of measurement (Chapter 11 and Appendix III). But mathematics has a deeper role than that of a language. As I shall try to show, our mathematics says more about the human race than is generally realized.

Some believe that there is no need to search. They believe that aliens come here often, and interact with people in strange, sometimes intimate, ways. Perhaps those who believe these things are responding in their own way to the needs many of us share. The need for a cosmos that is alive, for a cosmos in which we matter, for a cosmos in which there is warmth and love and room for good and evil. Anything but this cold, indifferent, expanding universe that science has shown us, this universe where we are the unlikely accident of evolution, alone in a vastness almost beyond our comprehension and doomed to remain in this remote location by the physics of relativity.

Late in the nineteenth century, and even into the early twentieth century, many thought we had found non-human company on the very next planet (Sagan and Shklovskii 1966: 275). This, of course, is Mars, the red planet, named for the Roman god of war. There are some striking similarities between this world and our own. The Martian period of rotation, its "day," is only about 37 minutes longer than that of Earth. The axis of rotation of each planet is tilted from the vertical—that of Earth by about 23 degrees, that of Mars by about 25 degrees. Hence both worlds have seasons. Like the Earth, Mars has bright, white polar caps that expand and contract with the seasons, and some darker areas of the planet undergo seasonal color changes. To many observers white polar caps meant water, and seasonal color changes meant vegetation. But reasoning by analogy like this, on Mars as it is on Earth, can lead to mistakes; we don't see what's really there, only what we think should be there.

It was in 1877 that the Italian astronomer Giovanni Schiaparelli reported seeing *canali* on the Martian surface. The Italian word means "grooves" or "channels," but it was translated into the English word "canals." Canals, of course, are artificial waterways, the result of construction and as such had to have a constructor. The implications of this "fact" inspired an American, Percival Lowell, to carry out extensive observations of Mars. He founded an observatory in Flagstaff, Arizona for just this purpose. These observations convinced him that Mars was the "abode of life," and in his books he presented a romantic portrait of a dying world, a world that, because of its small size and hence weak gravity, was losing its water to outer space. Its inhabitants, in a desperate attempt to survive, had constructed a planet-wide system of canals designed to bring water from the polar caps to the populated temperate regions.

This poignant image of our neighbors captured the popular imagination and led to many wonderful science fiction books. Who hasn't heard of the classic *War of the Worlds* by H. G. Wells? There was also a series of books by Edgar Rice Burroughs (yes, the same man who gave us Tarzan) that took place on Mars, and much later Ray Bradbury wrote another classic, *The Martian Chronicles.*

But not everyone was convinced that we had neighbors and as time and science moved on we learned that the polar caps were mostly frozen carbon dioxide (dry ice) and that the color changes were caused by large-scale dust storms that periodically rage across the planet. Our space probes have shown us that Mars is a geologically fascinating world, a world that might harbor microbial life, but it never was the home of the civilization contemplated by Lowell, or that of the princess imagined by Burroughs or the telepathic race pictured by Bradbury.

Still, there was hope. We merely turned our eyes in the other direction, towards the Sun. There we have the planet Venus. Often called Earth's twin, she is a beautiful sight in the evening or early morning sky and our telescopes showed us that she is, tantalizingly, covered by dense clouds. Again we reasoned by analogy with Earth. Clouds mean water, and lots of clouds mean lots of water. Surely there was an exotic world under those clouds; a warm tropical paradise or maybe a steamy jungle somewhat reminiscent of our own Jurassic era. And maybe there were even men and women down there—mermen and mermaids perhaps, because a world with so much water might be mostly ocean.

But once again reality caught up with our musings and we learned the horrible truth. Venus is a hellish world, hot enough to melt lead, with an atmosphere of choking carbon dioxide. And the clouds? On Earth clouds consist of droplets of water, but on Venus they consist of droplets of sulfuric acid (Kaufmann 1994: 197–99)! Such are our immediate neighbors. We lie between a planet that resembles the biblical hell and a frozen wasteland that is periodically subject to worldwide dust storms.

By 2012 our unmanned probes had visited all the major bodies of the solar system. There is no alien civilization here and so, if we are to find one, we must seek it elsewhere. In the twenty-first century, unlike earlier times, elsewhere means "out there" among the stars.

Is SETI a valid scientific project, or are we wasting time searching for something that isn't there? And even if we found an alien society could we hope to communicate with it? Is mathematics a kind of universal "Rosetta Stone"?

It is often assumed that we would share a great deal of mathematics with an alien race, but this assumption is never examined very closely. I do that here. We shall see that there is good reason to believe that an alien race could learn our mathematics and, in doing so, they would learn something about the human race.

REMARK: Natural Numbers, Sets, and Subsets

Attempts at communicating with an alien society generally involve using the natural numbers (i.e., 1, 2, 3, 4, 5, and so on). Since any society we contact must have something like the radio telescope (the methods available to us at the present time limit the kind of society we can contact), it seems reasonable that such a society would know these numbers, and would also be familiar with the process of counting. This is, however, an assumption and if we contact a society that doesn't know these numbers, we might have considerable trouble communicating with its members. In the Remark in Chapter 3, we suggest that it might have been the day-night cycle that led humanity to discover (or devise) these numbers. The language developed by myself and Richard Oehrle starts with the natural numbers because we couldn't think of anything simpler (DeVito and Oehrle 1990).

It would seem that the members of any intelligent race must be able to recognize collections of objects that exist in their environment. We have lots of words for collections of objects. We call a collection of cows a herd, but we usually call a collection of sheep a flock. A collection of wolves is called a pack, while a collection of fish is called a school. We have many names for collections of birds. We speak of a brace of pheasants, a covey of quail, a parliament of owls, and a murder of ravens.

The terminology of mathematics is much simpler. Any well-defined collection of objects, whatever those objects may be, is called a set. The term "well-defined" means that it must be clear just what objects are in the set and what objects are not. Those objects that are in the set are called its members or its elements.

When some objects are collected together into a set, something new is created. It is often convenient to indicate that a set has been created by listing its elements between a pair of curly brackets. So the set consisting of the letters a, b, and c is denoted {a, b, c}, and the set of all natural numbers is denoted by {1, 2, 3, 4, . . .}, with the ellipsis indicating that the numbers continue "forever."

Sometimes all elements of a given set are also elements of a second set. When this is the case we say that the first set is a subset of the second. We also say that the first set is contained or included in the second. Obviously every set contains itself. The other subsets of a set are called proper subsets. So the set of all robins is a subset of the set of all birds, because every robin is a bird. Since there are lots of birds that are not robins, it is a proper subset.

Chapter 2

Naïve Questions

Just what is the nature of this universe in which we find ourselves? Virtually every culture, and every age, has had its "answer" to this question. Models of the universe are as old and as varied as humankind itself. One picture, popular among some of the scientists in Newton's day, held that space went on endlessly in every direction, and that the stars occupied fixed positions in this space. There was no beginning; the universe was, and had always been, as we now see it. This model when combined with Newton's law of gravity led to a remarkable conclusion. *There must be infinitely many stars!*

You see, if there were only finitely many stars, then their mutual gravitational attraction would cause them to all clump together.

Since this obviously hasn't happened, the gravitational attraction of any group of stars must be off-set by the attraction of those outside the group. So no matter how far out you go, in any direction, there had to be stars even further out. Thus there must be infinitely many stars scattered throughout space.

This comfortable, and seemingly reasonable, picture was badly shaken when, in the early 1800s, an amateur astronomer, a German named Heinrich Olbers, asked a naïve question: Why is the sky dark at night?

Why is that a problem? Well, if our model was correct, then in every direction there would be a star. It would be like being in a forest where everywhere you look you see a tree, so the night sky should be as bright as the average star. So this simple question, now known as Olbers's paradox (although it was considered by another German, the great Kepler, as early as 1610), demonstrates that the universe is more complex than suggested by our simple model. The existence of infinitely many stars became questionable.

Perhaps this was for the best because the paradoxical nature of the very concept of an infinite set had been demonstrated by Galileo. The scholars of his day asserted that there were more natural numbers, {1, 2, 3, . . .}, than perfect squares, {1, 4, 9, 16, 25, . . .}. The square of a number is the number times itself.

Their reasoning went like this: Every perfect square is clearly a natural number, so the squares are a part of the natural numbers; they are a subset of the set of natural numbers. But, many natural numbers, like 3, 5, 7, and 8, are not squares, so the squares do not contain all natural numbers; they are a proper subset, not equal to the whole. Obviously the whole is always greater than any of its proper parts, so there are more natural numbers than there are squares.

Galileo had a character in one of his books present this argument. Another character, one perhaps representing Galileo himself, pointed out the flaw. Both collections are infinite so we are in the same position as the elders of an ancient clan, long before counting was invented, asking if they had enough spears to equip a hunting party.

All they had to do was have each hunter pick up a spear. If each man is armed and there are spears left over, they have plenty of weapons. If all the spears are taken and some men are empty-handed, they have an equipment shortage. And, of course, if each man is armed and there are no spears left, then the two collections, hunters and spears, are in one-to-one correspondence; they are equi-numerous.

This device was used by ancient peoples throughout the world to keep track of their herds or even their armies by setting up such correspondences between these collections and the notches on a stick or the pebbles in a pile. The words *tally* and *calculate* come from the Latin *talea,* cutting, and *calculus,* stone.

Galileo noted that the two collections, natural numbers and squares, can be put in one-to-one correspondence: 1–1, 2–4, 3–9, 4–16, and so on, so how can we say one is larger than the other?

In connection with SETI there is a paradox that, like Olbers's, centers on a naïve question. This one was asked by Enrico Fermi, winner of the Noble prize for physics in 1938. After collecting his prize he, together with his family, immigrated to the United States. He was concerned about the rising political radicalism then happening in Europe, especially since his wife, Laura, was Jewish. He taught at Columbia University then at the University of Chicago, and, during World War II, he was involved in the Manhattan Project.

Fermi was the man who, in a squash court under the stands of the athletic stadium at the University of Chicago, carried out the first sustained nuclear reaction. This was during the war (in 1942) and the director of the project, Arthur Compton, informed the Office of Scientific Research and Development of Fermi's success with the cryptic message: "The Italian Navigator has reached the New World."

In order to understand why the question now known as "Fermi's Paradox" arose, we have to look at some rather eerie events that were happening around the time it was asked.

It was shortly after World War II that the idea of alien intelligence, even the possibility of such intelligence visiting the Earth, thrust itself once again into the public consciousness.

The initial spark was a curious incident that happened in the summer of 1947. On 24 June of that year a businessman and experienced pilot, Kenneth Arnold, was flying over the Cascade Mountain range. He was looking for a lost Marine transport plane. Arnold never did find the missing plane but what he saw that day soon had much of the nation watching the skies. Upon landing at Pendleton Oregon, he told a local reporter that he had seen nine strange aircraft flying in the vicinity of Mount Rainier.

Arnold knew the area well and measured the time it took for the objects to fly from one mountain peak to another. He then used the known distance between the peaks and his time measurement to calculate their speed. It turned out to be far faster than any airplane then in existence. When asked how the objects moved, Arnold said, "As a saucer would if you skipped it over water" (Jacobs 1975: 36–38).

It was a slow news day and the story was picked up by the Associated Press wire service, giving it national attention. Odd stories like this are usually quickly forgotten, but that didn't happen in this case. Arnold's description of how the objects moved somehow became a description of the objects themselves.

Thus began the modern era of "flying saucers" and reports soon started coming in from all over the country. It wasn't only saucer or disk-shaped objects that were seen, but the term "flying saucer" caught the public imagination of the time. It seems to have been in the military, with its love for acronyms, that the more accurate phrase "unidentified flying object" (UFO) was first used.

Reports of strange objects in the sky are certainly not new (Vallee 1965). Throughout history there have been reports of weird things "up there." But, somehow, that sighting in June of 1947 opened a new facet of human consciousness. People saw UFOs, talked about UFOs, read about UFOs, and many came to accept them as part of reality.

Fads come and go. Who remembers the hula hoop or the cabbage patch doll? But UFOs have never left us. It is hard to find someone who has never heard of them, and most people seem to have some opinion about them. Perhaps something about the UFO resonates with the human psyche. The psychiatrist Carl Jung thought so (Jung 1959), but I think we must also consider the social and historical context in which the early sightings were made.

Shortly after the Arnold sighting many assumed that the saucers were an American secret weapon while others, those with perhaps a more paranoid turn of mind, thought that they might belong to some foreign power (Vallee 1965: 48–49). World War II was still a very recent memory. The terror of the V-2 rocket and the shocking power of the atomic bomb still lingered in the minds of many people. Who knew what other awesome developments were yet to be revealed? Moreover, the Cold War between the United States and the Soviet Union was getting nasty, and everyone knew that both sides had "acquired" German scientists skilled in rocketry. Perhaps they were behind the rash of sightings.

But by 1950 much of the public speculation centered on the idea that these strange craft might be alien spaceships. The sightings did have a certain "alien aura" to them, and the belief that our use and testing of atomic weapons might have attracted inter-planetary attention became popular. Conditions on the other worlds of the solar system were only poorly understood and intelligent life on one of these bodies could not be ruled out. Many scoffed at this idea, and they have been proven right, but the evidence as it was then known was not conclusive. As late as 1966 some scientists seriously suggested that the moons of Mars might be artificial satellites (Sagan and Shklovskii 1966: 373). So it is understandable that many people were convinced that UFOs were spacecraft from Mars or Venus or some other world in our solar system. The entertainment industry took note of this and, in 1951, released the highly popular movie *The Day the Earth Stood Still.*

In the summer of 1950 the trash barrels in the parks of New York City were disappearing at an alarming rate. At the same time numerous UFO sighting were being reported. One cartoonist connected the incidents by showing aliens carting trash barrels into their spacecraft. This cartoon, in turn, sparked a discussion among four physicists on their way to lunch one day. The four were Enrico Fermi, Emil Konopinski, Edward Teller, and Herbert York. The place was Los Alamos, New Mexico, home of World War II's super-secret Manhattan Project, where the world's first atomic weapon was constructed.

As they walked they traded arguments and counter-arguments about the questions raised by the ubiquitous UFO sightings. Was inter-stellar travel possible? Perhaps! Is it possible to achieve velocities greater than that of light? Maybe someday! Are the UFOs spacecraft? Highly unlikely!

Once these four gentlemen arrived at the restaurant, a place called Fuller Lodge, the conversation turned to topics of more immediate interest. Then, in the middle of a discussion of a totally different subject, Fermi suddenly asked, "Don't you ever wonder where everybody is?"

There was general laughter at the irrelevance of the question to the topic they were talking about. Fermi, however, was well-known for coming up with provocative questions (many of these "Fermi questions" are available online), and his companions realized that he was referring to extraterrestrials. They also realized that the question was more profound and troubling than it might at first appear. He then made a series of calculations from which he concluded that we ought to have been visited long ago and many times over. It would be interesting to know how he arrived at this conclusion but, unfortunately, his calculations were discarded.

The appeal of this paradox is, perhaps, that it succinctly summarizes three troubling observations. First, if, as many scientists believe, there are lots of scientifically sophisticated societies in our galaxy, and if, as many scientists believe, lots of these societies are much more advanced in science and technology than we are, then more than a few must have the ability to explore or even colonize the galaxy. We would expect, then, that some of them would have come this way sometime in human memory. Yet we have had no such visits!

Secondly, during many decades of extensive observations of outer space our astronomers have seen no evidence of technological activity at all.

Finally, radio astronomers have been listening to the stars for more than sixty years. In all that time nothing like an intelligent signal has been heard.

This absence of evidence is sometimes referred to as the "Great Silence," and while it may be true that "absence of evidence is not evidence of absence," we can't help but wonder, along with Fermi, "Where is everybody?"

As one might expect, Fermi's question has led to much speculation, lots of argument and counter-argument, and sometimes heated debate. There is a whole book devoted to these arguments (Webb 2002). Some say that the only reasonable answer to the paradox is that we are alone—there are no alien societies (Hart 1995). Others strongly disagree. "The argument for the non-existence of intelligent life is one of the most curious I have ever encountered," says one writer. "[I]t seems a bit like a ten-year-old child deciding that sex is a myth because he has yet to encounter it" (Webb 2002: 24).

Some say that intelligent aliens, if they existed, would already be here. In the abstract of one paper we find the statement: "It is argued that if extraterrestrial intelligent beings exist, then their spaceships must already be present in our Solar System." The author contends, as do others, that such beings would use self-replicating probes to explore and colonize the galaxy in a very, by cosmic standards, short time (Webb 2002: 24).

In response to this contention the late Stephen Gould wrote: "I must confess that I simply don't know how to react to such arguments. I have enough trouble predicting the plans and reactions of people closest to me. I am usually baffled by the thoughts and accomplishments of humans in different cultures. I'll be damned if I can state with certainty what some extraterrestrial source of intelligence might do" (Webb 2002: 24).

Scientists see the debate over Fermi's question as an example of scientific openness. There are lots of people, however, who don't see it this way at all. To them there is no paradox, and the entire debate is the result of an arrogant unwillingness to acknowledge the obvious answer.

Their thinking goes something like this: Fermi and his colleagues were discussing space travel because of the media attention given at that time to UFO reports. Isn't it obvious from the descriptions of these objects given by witnesses that many of them are alien spacecraft? And does it not follow that the pilots of these devices are intelligent and far ahead of us technologically? So there are aliens and they are visiting us right now! Why didn't Fermi and his companions notice this obvious answer to his question?

These are questions that, in the minds of many people, are quite reasonable. Questions that, many believe, have not been adequately answered by the scientific community. You will hear them raised after any public talk about SETI, and you will hear them asked on any radio or television program that deals with SETI.

Unfortunately these questions, so easy to ask, are not so easy to answer. An intelligent response to any of them requires a lengthy discussion that, to a great many people, sounds like a long-winded attempt to muddy the issues and evade the question. The reluctance of many scientists to believe UFO reports is *not* due to arrogance or an unwillingness to face facts.

It is rather based on hard-won facts about the universe we inhabit, and an understanding of how easily we can jump to unwarranted conclusions. Anecdotal evidence is suspect even in something as mundane as a minor traffic accident. It is certainly suspect when the witness talks about seeing a spacecraft or meeting with an alien, and the "evidence" for UFOs is almost entirely anecdotal.

The needs spoken of above (Chapter 1) may help explain why so many are so quick to believe reports of this kind. These needs, and the testimony of a sincere witness, must, of course, be respected. Ridicule and snide remarks are not the proper response, but neither is abandoning all reason and uncritically accepting any story that comes your way. As someone once said, it is good to have an open mind, but not so open that your brain falls out.

Sometimes the best you can do is to acknowledge that a story is interesting and maybe significant but suspend belief until more facts or other supporting evidence is forthcoming.

REMARK: Infinite Sets, Correspondences, Unions, and Intersections

Galileo's paradox is striking because the set of all squares is a proper subset of the set of natural numbers. Moreover, they are so spread out as to appear much smaller than the whole set. There are two things that should be mentioned here.

First, it can be shown that a set is infinite if, and only if, it can be put in one-to-one correspondence with one of its proper subsets. So this weird property is characteristic of infinite sets.

It may seem that any two infinite sets can be placed in one-to-one correspondence; you just pair up the elements and, since they are infinite, neither runs out. This is false!

It was Georg Cantor (1845–1918) who first discovered that, while it may seen paradoxical, there are sets that are larger—more "infinite"—than the natural numbers. In fact, given any infinite set, there is always an even greater set. Those interested can find a more systematic discussion of these matters in Appendix I.

By using the basic facts about sets of natural numbers one can communicate the so-called logical connectives. Given two sets, say A and B, we can combine them to form two new sets $A \cap B$ and $A \cup B$. The first of these is called the intersection of the sets and consists of all objects that are in both sets. The second is called the union and consists of all objects in one or the other of these sets. When you ask people if they want coffee or tea you don't expect them to say "both." The word "or" in that case is used in the exclusive sense. In mathematics, however, the "or" is used in the inclusive sense, so the intersection is a subset of the union. These constructions can be used to communicate the idea of "and" and "or." Similar constructions can communicate "implication" and "logical equivalence" and can form the basis for a simple language. The details can be found in DeVito and Oehrle (1990) and in Appendix III. By working with sets in a little more detail one can communicate the logical quantifiers: "for all" and "there is."

We might note that for any two sets A and B, $A \cap B$ is a subset of A (it is, of course, also a subset of B). It can happen that A and B have nothing in common, in which case $A \cap B$ is the empty set (when this is the case we say that the two sets are disjoint). Thus the empty set, the set with no members, is a subset of every set. This follows logically from the definition. Given any set A, every element of the empty set (there aren't any) is also an element of A. The empty set is so useful it has a special symbol: Ø.

Chapter 3

Are We Special?

One answer to the Fermi paradox is the simple assertion that we are alone in the universe or, at least, in the Milky Way galaxy. Could this be? There is, of course, no easy answer to this. But we can examine the Earth and the other planets of the solar system and note if, in any way, the Earth is unique.

Even the most superficial such comparison shows two things. First, there is an awful lot of water on our world. It covers nearly three quarters of the globe, and no other planet has anywhere near as much. Secondly, the Moon is unusually large in comparison to the Earth. The Earth-Moon pair is unique in our solar system. These two unusual attributes have some intriguing implications.

It is believed by many scientists that life first arose in the water. Many chemicals found their way into solution, and the inter-mixing of these ingredients eventually led to living organisms.

But what caused the mixing? Some say it was the tides, particularly the extreme spring and neap tides, which occur when the Sun and Moon are in line and when these two bodies are at right angles to each other. If this is so, then the Moon may have played a critical role in the origin of life on this planet.

It is a long way, of course, from simple aquatic life forms to humanity, and the process of evolution leading to us took many millions of years. We have seen that the seasons are caused by the fact that the axis of the Earth is tilted from the vertical (Chapter 1). Computer simulations show that the Moon has the effect of stabilizing the Earth's axial tilt over a period of many millions of years. This is important because even small changes in the angle of tilt can lead to dramatic changes in a planet's climate, and this can have a devastating effect on any ecosystem that may be present. Long-term stability of the Earth's climate gave evolution the time needed to produce the bio-diversity leading to the extensive ecosystem of which we are a part. But how did the Moon come to be here? There is an interesting theory about that.

Early in our Earth's history, before the asteroids settled into their orbits, impacts like the one that, many believe, killed the dinosaurs may have been very frequent. It has been suggested that the biggest collision of them all occurred 4.5 billion years ago. Our planet, then in the late stages of its own formation, was struck a glancing blow by an asteroid about the size of Mars. The collision shaved off a large slice of the Earth's surface, knocking it into space. Much of the debris, liquefied by the impact if not already molten, entered into orbit, was cooled, and then reconstituted as our Moon. This event is known as the "big splat." It produced our Moon, tilted the Earth relative to its plane of rotation thereby causing our seasons, and contributed to the regular alternation of nights and days by affecting our planet's spin (Webb 2002: 185–89).

The lives of many animals are tied to the cycles caused by the big splat. Some sleep at night, others during the day, the rhythm of their lives in sync with the daily cycle. Many birds and even herds of large animals migrate seasonally. And there are some whose lives are in tune with the more complicated variation of the tides.

The grunion, a small fish, lays its eggs far up on the shore during spring tide. There the eggs remain undisturbed until the next spring tide, but that's exactly when the eggs hatch and the hatchlings, suddenly immersed in water, can swim out to sea. Shellfish, gathered on the west coast and transported to the Midwest, will continue to open at the time of high tide—high tide in the waters where they were gathered. We shall see how some people cleverly exploit these biological connections to the Earth's astronomical cycles.

The cycles had their effect on human life as well. Many early societies were dependent on the seasonal migration of large herd animals. The Native Americans of the plains, for example, relied on the annual appearance of the bison herds. This is why some, who wanted to subdue the Native Peoples and take their land, slaughtered these animals almost to the point of extinction.

With the development of agriculture, the seasonal cycle became vitally important. But the cycles may have had an important role in human intellectual development. They may have taught us to count.

We have already noted how early people set up one-to-one correspondences between collections of objects that interested them, like their herds or even their armies, and pebbles or notches on a stick. Here they were comparing cardinal numbers; the number of objects in a collection is called the cardinal number of that collection.

Comparing cardinals does not require counting. We just pair up the objects in the two collections until one runs out. But counting involves the other, more subtle, aspect of number. The numbers form an ordered sequence. There is a first, a second, a third, etc. Where did this idea come from? I think it came from the day-night cycle.

To early humans distance wasn't important since, unless two sites were very close together, they had no way of measuring it. What was important was the time it took to get from one site to another. This could easily be measured. The traveler could carry a stick and, at the end of each day's journey, carve a notch on the stick. But while the animals in a herd could be led before a carver in any order, the days come in a fixed order which must be followed; this was long before the days had names and were conveniently collected into weeks. You had to record them as they came, and, perhaps very slowly, it was recognized that two followed one, and was followed by three, and so on.

This is so well-known to us now that we are often unaware of the distinction between cardinal number, the number that really interests us, and ordinal number, the number that allows us to find the cardinal number. But it was a great leap in understanding for early humans and probably was discovered independently in many parts of the world; it is possible that it was discovered, forgotten, and rediscovered more than once.

The development of agriculture drew attention to the seasonal cycle, and to keep track of these changes people invented the calendar. Calendars were usually based on the Moon and, because of this, complications arose. Sometimes the number of days in a lunation, the period between one new moon and the next, is thirty and sometimes it is twenty-nine. In some years there are twelve new moons and in others there are thirteen. Adjustments had to be made. Different people came up with very different ways to do this.

The residents of Vakuta, in the Trobriand Islands, rely on the biological clock of a certain marine annelid. This creature spawns just once each tropical year, at the time of the full moon, in the seas off this island during the month they call Milamala.

If the worm does appear they begin a new year; if it does not appear at this time the month is repeated and their year has thirteen months (Ascher 2002: 43). Here is an example of a human society exploiting the connection between an organism and the seasonal cycle. In this way they keep their calendar in sync with the seasons and avoid the necessity of keeping records or making calculations.

But having to make calculations can be a very positive thing. It forces one to learn something about numbers. The Jewish people, who had no obliging organism in their environment, noted that 19 solar years is almost exactly 235 lunations (the discrepancy is about 4.5 hours). They also noted that the number 235 is 12 times 12, plus 7 times 13. So they arranged their calendar to have 12 years with 12 months, and 7 years with 13 months.

Notice that the Jewish calendar has an imposed nineteen-year cycle. Other people imposed cycles on their calendars as well. Some of these were for convenience, like the week, others for social reasons, like the Roman fifteen-year taxation cycle.

These often led to problems of arithmetic leading the societies involved to investigate numbers more closely and develop a deeper understanding of their properties. The week imposes a seven-day cycle on our calendar. This cycle forces the numbers assigned to a particular day, in any month, to be "congruent modulo seven"; this means that the difference between any two of these numbers is a multiple of seven. The Mondays in October of 2012, for example, fell on the 1^{st}, 8^{th}, 15^{th}, 22^{nd}, and 29^{th}.

The ancient Mayans imposed two important cycles on their calendar, one with a period of thirteen days and another with a period of twenty days. This led them to seek numbers that were congruent modulo thirteen (the difference of any two of these numbers is a multiple of thirteen) and also congruent modulo twenty. Problems of this kind were also considered by the ancient Chinese. In fact their scholars found a mathematical result now known as "The Chinese Remainder Theorem" (Dickson 1957: 11).

Collisions between bodies in an early planetary system may be quite common. But collisions that lead to one of the planets having a large moon may be very rare. However our Moon came to exist, it seems that it had a role in the development of life, and a role in keeping the seasons stable so that that life could evolve over a long period of time. Whether or not this would always lead to intelligent life is unknown, but that is what happened here. As we indicated above, the Earth-Moon-Sun system, with its fascinating cycles, may have played a crucial role in human intellectual development. There are plenty of stars in the galaxy and, we are learning, plenty of planets, too. But how many of those planets have lots of water, are located in a region where much of the water can remain liquid (see Chapter 8), and have a large moon to stabilize their seasons, giving life a chance to form and evolve?

There is one other process that is crucial in regulating the Earth's climate and requires the presence of water. This is called "plate tectonics": the sliding of oceanic plates, deep under ground, under continental plates. Plate tectonics is at the heart of the carbon dioxide recycling loop, and it is water that allows the crustal plates to glide over the hot mantel rocks. Were this process to stop two things could happen.

Either most of the carbon dioxide would remain in the atmosphere, leading to a run-away greenhouse effect as we see on the planet Venus, or it becomes locked in the ground in the form of minerals, leading to a freeze similar to that found on the planet Mars (Darling 2001: 78–79).

The day-night, lunar, and seasonal cycles are experienced by all—not just by people but by animals as well. And yet, although many animals seem to have a rudimentary number sense, none have learned to count. We are the only ones on Earth who create systems of mathematics that can model aspects of the cosmos, and we are the only ones who build radio telescopes. Among all the inhabitants of Earth, we are, at least in this limited sense, special. We have studied the universe extensively. We have learned something of its structure, its grandeur, its vastness. This is no small accomplishment. But why do we do it? My colleagues in the social sciences are the people best qualified to answer this question. Let me just quote an early "answer" that might provoke some discussion.

In the early twentieth century Fridt Jof Ansen, using the somewhat sexist language of his time, explained it like this: "The history of the human race is a continuous struggle from darkness toward light. It is therefore of no purpose to discuss the use of knowledge—man wants to know and when he ceases to do so he is no longer man. . . ."

The fact that we desire to know, especially to know about the universe in which we find ourselves, is a deep, perhaps defining, aspect of our humanity. It shows that underlying our warlike and destructive ways there may be a spark of real intelligence.

REMARK: Systems of Enumeration, Powers of Ten, Positional Notation, and Casting Our Nines

The names we give to numbers vary from country to country, and, throughout history, numbers have been symbolized in many ways. The current, pretty much universal, system for writing numbers is based on the number ten.

The number ten is convenient for several reasons. The symbol 10^n means we are to multiply 10 by itself n times. This is easy to do: just write 1 followed by n zeros. So 10^3 is just 1,000, and 10^9 is just 1,000,000,000.

This enables us to write very large numbers more conveniently. The number of stars in our galaxy is estimated to be four hundred billion, or 4×10^{11}. Given a sample of, say, carbon, we can measure its weight. We can't, of course, count the number of atoms in the sample. This can be calculated from the weight using the Avogadro number 6.023×10^{23}. Imagine writing this out in its entirety.

We can also conveniently write very small numbers using powers of ten, this time negative powers. The symbol 10^{-n} means $1/10^n$. So 10^{-2} is $1/10^2$ or 0.01, and 10^{-3} is $1/10^3$ or 0.001, and so on. An important number in physics is Planck's constant, which is 6.626×10^{-34}.

We can write any whole number by using just ten symbols, the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. This trick is called positional notation. In the number 1,111 it is understood that the leftmost digit is one thousand, the next is one hundred, next is ten and, finally, the last one really is a one: 1,111 = 1,000 + 100 + 10 +1, or $10^3 + 10^2 + 10 + 1$. Similarly, 4,357 is 4,000 + 300 + 50 + 7, or $4(10^3) + 3(10^2) + 5(10) + 7$.

The number ten is the base of our system of enumeration. Of course it is certainly possible to use some other natural number, except the number one, as our base. This is often done in computer science for example. Other bases were used by human societies. Some used twenty (presumably they counted on their fingers and toes), and some used sixty. Vestiges of this are found in the way we measure time (sixty seconds in a minute, sixty minutes in an hour), and in how we measure angles (one degree contains sixty minutes, and one minute contains sixty seconds of arc).

Since the number ten is the base of our number system, when we subtract from any number the sum of its digits, we always get a multiple of nine (one less than the base). So, for example, the digits in 38 add up to 11, and 38 minus 11 is 27 which, of course, is 9 times 3. The digits in 1,221 add up to 6, and 1,221 minus 6 is 1215, which is 9 times 135.

In the terminology introduced above, any whole number written in base ten is congruent to the sum of its digits modulo nine. This can be used as a quick check on one's arithmetic and used to be taught as "casting out nines."

Adding 111 and 11 gives us 122. The digits in 111 add to 3, and those in 11 add to 2, so the digits in our sum, if we did it right, should add to 2 plus 3. This works for multiplication as well. The product of 111 and 11 is 1,221. If we did this right, the digits in this number should add to 2 times 3.

We have no way, of course, of knowing how an alien race would write its numbers—assuming that they understand and use numbers. It seems to me that any society that has the radio telescope would understand the natural numbers and know how to count. If they use positional notation the base of their system may tell us something about them. Very large and very small numbers arise in many areas of science. Our correspondents must have some way of dealing with this. The scientific notation using powers of ten is our way, and it, or some variant of it, may be something we may share.

I should stress that in our system of enumeration the symbol 10 represents the number ten. In other systems this symbol represents the base of that system. So if we use eight as our base then the symbol 10 represents eight and the powers of ten, discussed above, must be understood to be powers of eight.

Chapter 4

Stories—Part One

As far back as anyone has been able to probe, via folklore or ancient writings, people have reported seeing strange objects in the sky. UFOs are not new (Vallee 1965: 1–24). Still the modern "incarnation" of the subject is usually said to have begun on 24 June 1947, when Kenneth Arnold made his sighting over Mt. Rainier (Chapter 2). Other sightings soon followed, most of these can be found in Peebles (1994). At Maxwell Air Base, in Montgomery, Alabama, several witnesses, including pilots and intelligence officers, watched a light streak across the sky, make a *right angle turn,* and then disappear. This was on 28 June 1947, and the next day several rocket scientists at White Sands, New Mexico saw a disk fly by at a speed that, they estimated, exceeded that of sound; this was a few months before Chuck Yeager broke the sound barrier.

On 4 July there were many reports, from Portland, Oregon, of flying disks seen by police officers, harbor patrol men, and others. At 9:12 PM that same day the captain, his co-pilot, and a stewardess on United Airlines Flight 105 saw five disks flying in formation. These flew off suddenly only to be replaced by four more. The entire sighting lasted ten minuets.

*The New York Time*s of 6 July carried a list of possible explanations for what people were seeing (Peebles 1994: 10). Among them was this provocative statement: "They may be visitants from another planet launched from spaceships anchored above the stratosphere."

On 8 July technicians observing an ejection seat test at White Sands saw a metallic object suddenly come into view, fall nearly to the ground, and then rise again and vanish.

The disks seemed to be everywhere, and very competent people were reporting them. People were alarmed, or at least, concerned. Were we under surveillance by an alien race? If so, what would they do next?

Most disturbing were the flight characteristics of the disks. They could fly at speeds far faster than those attainable by our best planes, and they were far more maneuverable. The disks were in control. We didn't know where they were from, who was flying them, or what they wanted. And then the word spread that the Air Force, then a branch of the Army, had got hold of one!

The press release that stunned the world came from a remote corner of the United States, a place called Roswell, New Mexico. It was issued by Lt. Walter Haut, the information officer at Roswell Army Air Field, on 8 July 1947. It read:

> The many rumors regarding the flying disc became a reality yesterday when the intelligence officer of the 509th Bomb Group of the Eighth Air Force, Roswell Army Air Field, was fortunate enough to gain possession of a disc through the cooperation of one of the local ranchers and the sheriff's office of Chaves County.
>
> The flying object landed on a ranch near Roswell sometime last week. Not having phone facilities, the rancher stored the disc until such time as he was able to contact the sheriff's office, who in turn notified Major Jesse A. Marcel of the 509th Bomb Group Intelligence Office.
>
> Action was immediately taken and the disc was picked up at the rancher's home. It was inspected at Roswell Army Air Field and subsequently loaned by Major Marcel to higher headquarters. (Peebles 1994: 247)

It wasn't a disk that was found, but debris that some thought might have come from a damaged craft that touched down in the desert before flying off again. Some of that debris was collected by Major Marcel and was taken by him to Carswell Army Air Field at Fort Worth, Texas. The material was then taken to the office of General Roger Ramey. Marcel was ordered into the map room and told to pinpoint, for the general, the spot where the material was found.

Upon returning to the office the press was invited in to take pictures of Major Marcel, General Ramey, and Colonel Thomas DuBose, the general's aid, examining the wreckage. The base weather officer was also brought in, and he immediately identified the material as part of a Rawin Target weather balloon.

Marcel would claim later that a switch had taken place; the weather balloon fragments had been substituted for the material he had brought with him from Roswell. At the time of the press meeting, however, he was under orders to remain silent.

Many years later the Air Force, now a separate branch of the armed forces, admitted that a switch had taken place and that the weather balloon story was, in fact, a cover-up (see Chapter 9).

Perhaps the most amazing aspect of the cover-up is the reaction of the news media. The soldiers at Roswell were an elite group. They were the only group in charge of storing and, if necessary, delivering America's atomic weapons.

Somehow no one questioned how the intelligence officer and others at the base were unable to recognize a common weather balloon. This incident, which looms so large today, simply faded away and was forgotten for several decades.

Although the Roswell story quickly faded from public consciousness, the flying disks did not. They were still being seen, sometimes by trained observers over areas of military importance like White Sands Missile Range. People wanted answers and no one seemed to have any. Concern increased when a dramatic sighting made the headlines of the nation's newspapers. This time someone got close to a flying disk, and that someone died (Peebles 1994: 18).

There were numerous reports of a spherical object in the skies over Kentucky on 7 January 1948. The object was moving slowly south, and witnesses on the ground estimated its diameter to be between 250 and 300 feet.

As it happened four aircraft were approaching Godman Air Force Base while the sphere was in view. Those in the base control tower asked the flight leader, Captain Thomas Mantell, to investigate. He agreed, and with him in the lead, three of the planes went after the UFO. The fourth plane was low on fuel and did not participate in the chase.

None of the planes was equipped with oxygen, and, as I've been told by military pilots, when flying such a plane, one does not go above 12,000 feet. Yet, both Air Force Captain Edward Ruppelt and aerospace historian Curtis Pebbles state that Mantell was at 15,000 feet when he radioed the tower and reported that he clearly saw the object directly ahead of him. When asked to describe the UFO he said, "It appears to be a metallic object or possibly reflection of Sun from a metallic object, and it is of tremendous size." He next said, "I'm still climbing, the object is above and ahead of me moving at about my speed or faster. I'm trying to close in for a better look."

Nothing more was heard from him and a short time later his plane crashed near the town of Franklyn, Kentucky. It was conjectured that he had flown too high and, due to lack of oxygen, blacked out. His plane went into a dive and there is some evidence that he did regain consciousness and try to save himself. But, tragically, it was too late.

An incident like this generates a great many rumors. Some said the plane was riddled with bullets. It wasn't! Some said Mantell's body was never found. It was! Others said the wreckage was "radio-active"; in those days this word was scary and hinted at a dangerous mystery. And still others said the plane had been "magnetized" (see below). None of this was true.

But the incident made the public uneasy; no one knew what to believe. What had really happened to Mantell, and what was it that he was chasing? The Air Force's explanation did little to ease the public's concern. They said Mantell was chasing the planet Venus!

There is no doubt that Venus has been the stimulus for a great many UFO reports (Craig 1995: ch. 4), but on that bright, clear day, at that time, 3:15 in the afternoon, Venus was a point of light, hard to see unless you knew exactly where to look. No one would claim, "It is of tremendous size." This cavalier and rather dismissive explanation of so tragic an incident did not enhance the public image of the Air Force, nor did it enhance public confidence in the way it was handling the UFO phenomenon.

The cause of the tragedy, or perhaps I should say a more plausible explanation of what the various witnesses saw, was found by Captain Edward J. Ruppelt when he was head of the Air Force's Project Blue Book. This wasn't until 1952 however, and by then the idea that a flying saucer had shot down an Air Force plane was firmly fixed in many minds. According to Ruppelt, Mantell had been chasing a (then top secret) Skyhook balloon that had been launched from Camp Ripley, Minnesota, early on the morning of the crash (Peebles 1994: 56–57).

These were huge things, 100 feet in diameter, and could reach a height of 60,000 feet. They could move, depending on the winds aloft, at up to 200 miles per hour. Those involved in the sighting knew nothing about these balloons, and those in the Air Force who did were unable to speak out.

There is one thing lacking from Ruppelt's explanation. Why would an experienced pilot like Mantell, who was known to be a cautious one not given to taking foolish chances, take his plane above 15,000 feet when he had no oxygen?

I have been told that a young person, in good physical condition, can fly at 15,000 feet for awhile. But, at this height, one's judgment soon deteriorates. Maybe this is what happened to Mantell.

The idea that the Mantell tragedy might have something to do with "magnetism" comes from a book "Behind the Flying Saucers" published in September of 1950. This book, written by Frank Scully, tells of three flying saucers that crashed in the southwest and were recovered by the government.

According to Scully the first of these, which contained sixteen bodies, landed near Aztec, New Mexico, while the other two landed in Arizona. One of the latter also contained sixteen bodies while the other much smaller craft contained only two.

The book sold 60,000 copies in hardback, was serialized in a magazine and also came out in paperback (Peebles 1994: 67). Scully got his information from Silas M. Newton, a Denver businessman, and from a rather shadowy "Dr. Gee" who, it was claimed, was the top magnetic specialist in the United States. According to this distinguished scientist the saucers used some kind of magnetic propulsion system which also enabled them to destroy an object simply by "demagnetizing" it. This was, in fact, how Mantell had met his end.

The bodies found in the craft were all between 36 and 42 inches tall, well-proportioned, and with perfect teeth that had no cavities or fillings. The saucers were intact, although one had a broken porthole, and all three probably came from the planet Venus. Thus the reality of UFOs was at last firmly established, and their probable origin now known.

It was a great story but, unfortunately, an investigative reporter, J. P. Cahn, looked into Scully's claims (Peebles 1994: 67–71). He found that the mysterious Dr. Gee was Leo GeBauer, operator of a radio parts store in Phoenix. Silas Newton had a reputation for questionable business practices, and the entire story was a hoax. Both men were arrested by FBI agents for selling a device that, it was claimed, could detect oil deposits and, within a year, they were convicted of conducting and conspiring to commit a confidence game (Peebles 1994: 70–71). Scully was a columnist for the show business magazine *Variety*. He may very well have been a victim of the two men, although he never admitted it (Peebles 1994: 71); he claimed that Dr. Gee was a composite of several scientists he had met and that he invented the character to preserve their anonymity.

REMARK: Human Perception of Motion, and Mathematical Description of Physical Fields

There are many reasons why the scientific community was highly skeptical of the reports of unidentified flying objects. For one thing, UFOs seemed to violate the laws of motion. According to witnesses these objects could make right angle turns while flying at tremendous speeds, and accelerate from rest so fast that they almost seemed to disappear; sometimes it was reported that they actually did disappear "like a light that was simply turned off."

We perceive motion as a continuous process, and the behavior described seems to violate that perception. Our perception is not to be disparaged or ignored because by making it precise we have been able to make considerable progress in physics and astronomy. The mathematics of motion is differential calculus, and unraveling its intricacies took many decades. We shall have occasion to discuss this further in subsequent chapters (Chapters 9, 10, and

16). It wasn't until the nineteenth century that mathematicians realized that the foundations of calculus had to be based on a deep understanding of the properties of the real number system.

Recognition that such a system existed first arose in connection with geometry (see Chapter 15). To begin the discussion first recall that the natural numbers are contained in a larger set called the set of integers. This consists of all natural numbers, their negatives, and the number zero, and is usually denoted by Z (from *Zahlen,* the German word for number). So N = {1, 2, 3, 4, . . .} and Z = {. . .–4, –3, –2, –1, 0, 1, 2, 3, . . .}.

Incidentally we might note that there is a one-to-one correspondence between N and Z. When the natural number n is even we let it correspond to n/2. So 2 corresponds to 1, 4 corresponds to 2, 6 corresponds to 3, and so on. For the odd numbers we let the natural number 1 correspond to 0, 3 to –1, 5 to –2, 7 to –3, and so on. Here again we see an example of an infinite set, this time Z, in one-to-one correspondence with one of its proper subsets.

The integers are part of a larger set of numbers called the rational numbers. These are all numbers that can be written as a quotient, or ratio, of two integers—excluding, of course, zero in the denominator.

So all the integers, and numbers like 1/3, 1/9, 1/16, 22/7, etc., are rational numbers. The entire set is denoted by the letter Q.

Now Q seems to be the "natural" end of this process. We can carry out all of the arithmetic operations in Q and the results, our "answers," are again in Q. Why go any further?

The first subtle intimation that there was a larger number system that contained Q came from an ancient Greek sailor. There was a philosophical school in ancient Greece, the Pythagoreans, whose members believed that all things could be expressed as a ratio of whole numbers. One day, while out at sea, one of their members discovered that the famous theorem of Pythagoras (the founder of their school) implied that there are numbers that are not rational (his proof is given in Chapter 15 and the theorem is also stated there). Excited by his discovery he rushed to show his shipmates his proof. They too were excited, but not in the same way he was. According to legend, they arrived back at shore missing one crewman.

According to Scully's very dubious sources, the flying saucers were propelled by magnetism. We shall have occasion to discuss this phenomenon again in another chapter (Chapter 6).

An illuminating picture of the space around a magnet was introduced in the nineteenth century. This is based on the idea of a correspondence between two sets but, in this connection, the sets involved are not sets of numbers. We all know that a magnet will attract a piece of iron placed in its vicinity. This requires a force exerted by the magnet on the iron. In order to quantify this, physicists introduced the idea of a magnetic field. At each point in the space around a magnet we may associate an entity called a vector. A small compass placed at a given point near the magnet will experience a force that can be measured. The force has both a magnitude and a direction. So at each point around the magnet we can imagine an arrow, pointing in the direction of the force, whose length is determined by the strength of the force. The arrow is called a vector. In this way we associate to each point in the space surrounding the magnet a well defined arrow. Such an association is called a vector field. In a similar way we can associate a vector field with a charged particle. This, too, will exert a force on any test particle placed in its vicinity.

There are other kinds of fields besides magnetic or electrical. The Earth, for example, exerts a force on any material body in the space around it. This is a gravitational field. It, too, can be quantified and depicted as a vector field.

Magnetic fields can be exploited to generate electricity. If a coil of wire is rotated in a magnetic field a current is produced in that wire. This is, of course, the principle on which the dynamo is based. Coils of wire placed within a magnetic field are rotated by water or wind, and the electricity produced is carried off for home or industrial use.

Chapter 5

Measuring Our Solar Neighborhood

The sun emits a prodigious amount of energy, but it doesn't do this at a constant rate. Many phenomena in the solar atmosphere go through an eleven-year cycle (Kaufmann 1994: 316). So, back in 1952, the members of the International Council of Scientific Unions could predict that the period from 1 July 1957 to 31 December 1958 would be one of peak solar activity. They designated this period the International Geophysical Year and suggested, among many other projects, that artificial satellites be launched during this time to map the Earth's surface. In response to this, the White House, in July 1955, announced plans for an Earth-orbiting satellite and solicited proposals for such a satellite from various government research facilities.

Unfortunately there was conflict and competition among the nation's military services (Army, Navy and Air Force; the Marines were concerned with other matters) which wasted time and energy. The contract finally went to the Naval Research Laboratory for its Vanguard project.

The ever secretive Soviets however, were not sitting still as the world was to learn on 4 October 1957. On that day, under the leadership of Sergei Pavlovitch Korolev, they made history by launching Sputnik One, the world's first artificial satellite. This was about the size of a basketball, weighted 183 pounds and orbited the Earth every ninety-eight minutes. It broadcast a "beep" to the world below.

This spectacular demonstration of Soviet technology surprised, amazed, and, yes, frightened much of the world. If they could do this, then why couldn't they place a ballistic missile anywhere they wanted? Remember this was deep in the Cold War and, to the western world, the motives of the Soviets were always suspect.

The question was raised again, with greater urgency, one month later. On 3 November, Korolev launched Sputnik Two. This weighed 1,100 pounds and carried a live dog named "Laika." What would they do next? And what was America, the self-styled leader of the free world, doing to match this? Unfortunately the launch of Vanguard, which everyone knew was anti-climatic, failed. The rocket blew up, and America's first satellite, which weighed a mere three pounds, never made it into orbit. The world soon referred to this catastrophe as "Kaputnik."

Americans, however, are always quick to respond to a challenge. After the bombing of Pearl Harbor, young people rushed to join the armed forces or in other ways support the war effort. Now, after Sputnik, they rushed to enroll in engineering schools. Everyone wanted to be an engineer. Science, which until then had generally been considered weird, comical, and even un-American, was now suddenly okay. Space, something few had even thought about before, was now where the action was. Scientists previously dismissed as "egg heads" and "dreamers" (the term "nerd" hadn't been coined yet) were suddenly respected.

The Soviet Union achieved another spectacular first in 1961 when cosmonaut Yuri Gagarin orbited the Earth on 12 April. That year President John Kennedy challenged the Soviets to race us to the Moon; America had to do something to regain the world's respect. Ridicule of our space efforts, even from some of our allies, didn't really stop until we won that race in 1969. Then people started saying that going into space really wasn't important after all. I wonder what they would have said if we had lost that race?

But going to the Moon requires knowing just how far away that body is. How can we know something like that? After Sputnik some people asked, naïvely, how the Soviets knew it wouldn't hit a star! It is a real, non-trivial question: Just how do we know the distances between us and the various objects "out there"? To answer this question we must go back to ancient Egypt and discuss the birth of geometry.

Every year the Nile River floods its banks. This is a crucial event for those who farm along its shores since the waters not only irrigate the land but also deposit vital nutrients onto the soil. At some point the early sky watchers noticed that this life-giving event correlated with the rising of the star Sirius shortly before dawn. This must have seemed magical to these ancient people; a sign from the gods themselves and a clear demonstration that heaven and Earth were inter-connected. Here was a celestial event that signaled the onset of a terrestrial phenomenon of paramount importance; it was almost enough to make one believe in astrology. You can bet that the early astrologers tried to capitalize on it!

But the annual flooding had a downside. The waters obliterated the boundaries between adjacent farms, and the people were faced with the problem of how to correctly reset those boundaries after the water had receded. The rules of thumb that they devised were, perhaps, the first intimation that space has properties that can be usefully exploited.

Those involved, however, were thinking about concrete problems, not space in the abstract. No one pursued these matters until many years later when a Greek gentleman named Thales traveled to Egypt and learned about them. Impressed, and highly intelligent, he initiated a systematic study of the properties of space, giving us the world's first formal geometry. His work, and that of many other Greek scholars, was immortalized in a series of books written around 300 BCE by Euclid, and is now usually referred to as Euclidean geometry—a remarkable achievement and one of the great accomplishments of the ancient world.

Interestingly, archeological evidence uncovered in the twentieth century shows that the peoples of Mesopotamia, as early as 1,700 BCE, knew quite a bit about the properties of space. In particular, they knew the Pythagorean Theorem, a major result in Euclidean geometry, more than a thousand years before Pythagoras lived (Edwards 1984: 3). Apparently, geometric facts were known more widely, and known much earlier, than previously thought.

Now all of this is, perhaps, very interesting, but what does it have to do with the questions asked above? I've often heard it said that scientists like to bring up esoteric facts that have nothing to do with the questions they are asked, in this way confusing matters, so that they can avoid answering embarrassing questions. This isn't true. Science can be subtle, and sometimes seeing the connections between things can take some time and some patience. The geometry mentioned here is very relevant to the questions raised.

To begin to see the connection we must start with the problem of finding distances between the various planets of our solar system and the Sun. There is a very practical offshoot of geometry called trigonometry. Here one learns how, given the measure of some parts of a triangle, one can compute the measure of the other parts. If you know, for example, the length of two of its sides and the measure (number of degrees) of the angle between them, then you can calculate the length of the third side and the measure of the other two angles.

Well, so what? The point is that by cleverly constructing your triangle you can compute distances that cannot be measured directly; like the radius of the Earth or the distance from the Earth to the Moon.

There is some controversy about where this subject originated. Was it in India, or was it in the Islamic world (Arabia and Iran—then known as Persia)? No one seems to know. There is no doubt, however, that the full potential of the subject was developed by the Islamic scholars. They gave us the modern system of decimal enumeration so important in making these calculations, and they compiled the world's first, and for many decades the world's only, trigonometric tables; before the invention of the hand calculator, important numbers were placed in a list, sometimes called a "table." These matters are discussed further in Chapter 7 where we shall see that the ratio of the sides of a certain type of triangle does not depend on its size, only its "shape." These ratios, which are useful in solving many practical problems, where computed by the Islamic scholars.

The astronomers were quick to see the usefulness of trigonometry for their science, and it is here that the relevance of this subject to the questions raised above becomes clear.

In order to find the distance from a given planet to the Sun they constructed an imaginary triangle having one side be the distance they wanted to find and another side the distance from the Earth to the Sun. The latter distance is called 1 A.U., one astronomical unit. They then measured the angle between these two imaginary sides (this wasn't easy) and, using the tables, computed the desired distance in terms of the astronomical unit. In this way they found that Mars is at 1.5 A.U. from the Sun, Jupiter is at 5.2 A.U. from the Sun, and Neptune is at 30 astronomical units from the Sun. Pluto is even further out, but its orbit is so eccentric that it sometimes lies within that of Neptune. The planets rotate around the Sun in elliptical orbits. Every ellipse has an associated number called its eccentricity. This is always between zero and one. The closer it is to zero the more nearly circular the curve. The eccentricity of the Earth's orbit is only 0.017, so we can think of it as a circle.

Pluto is one of the major objects in the Kuiper Belt, a region starting at about 30 A.U. and ending at about 50 A.U. that is only now beginning to yield some of its secrets. The solar system extends far beyond this. The comets live in the Oordt Cloud which starts at about 10,000 A.U. and extends all the way out to 100,000 A.U.!

The crucial question now is just what is this A.U.? Just how far is the Earth from the Sun? It turns out to be about 93,000,000 miles (Kaufmann 1994: 10)! Ninety three million miles, and this is just the "yardstick" with which we measure distances in our immediate neighborhood. The Moon is one quarter of a million miles away, or 0.0027 A.U. This figure is, perhaps, the best answer to those who say, "We've gone there, so why can't they (the aliens) come here?" The fact is that humans have never gone very far off-planet, and the reason for that is that manned space travel is very, very difficult. Sending men to the Moon and retuning them safely was a remarkable achievement, more remarkable than many seem to realize. There is some talk now of sending men and women to Mars, at times about 0.5 A.U. from Earth—about 46,500,000 miles, which is quite a bit further than the distance to the Moon.

The trip alone, ignoring other difficulties like life-threatening radiation, will tax our technology to its limit. This is not to say we shouldn't try it, just to put the project in perspective.

Our neighboring planets are millions of miles away but the stars are far, far further than this. Going to the stars is not just the next step. Pictures of a star field, taken six months apart, show that the position of some of the stars has shifted by a small but still measurable amount. Since six months is half a year, and the Earth takes a full year to circle the Sun, these pictures give views of the star field from opposite ends of the diameter of the Earth's orbit (2 A.U.). Using a little geometry, the shift in position enables one to calculate the distances to these stars. The numbers obtained are so large that even the astronomical unit (93,000,000 miles, remember) is too small to conveniently represent them. Astronomers use the distance that light travels in one Earth year as their yardstick. To find this number, the light-year, one multiplies the speed of light, more than 186,000 miles per second, by the number of *seconds* in a year.

It works out to be about six trillion—6,000,000,000,000—miles (Kaufmann 1994: 10). The astronomical unit is less than *one sixty thousandth* of that!

The star nearest the Sun, Proxima Centurii, is about four light-years away—about 24,000,000,000,000 miles, and that's the nearest one! Furthermore, the method sketched above, called the method of parallax, works only for stars that are within 300 light-years from us. Most stars are much further away than this.

Now it should be clear why so many scientists are so skeptical when they are told that UFOs are the spaceships of inter-stellar visitors. Scientists are fully aware of the "astronomical facts of life." They know that, even at light speed, a journey from any star to the Earth would take years. Yet there have been literally thousands of reports by people who claimed that they saw UFOs. Is it really credible that thousands of aliens would make the immense journey to come here, and once here do little more than frighten a few of the natives?

Now, as is well known, time slows down for those traveling at near light speed (Chapter 14). The journey, for such travelers, could be quite short. But they'd miss out on a lot going on at their home planet because when they returned home, they'd find that everyone there had aged many years. If their home star is, say, thirty light-years away, then when they come to Earth and then return home, at least sixty years will have passed on their home world.

REMARK: Euclid's Fifth Postulate, Non-Euclidean Geometries, and How Choice of Geometry Affects Physics

Euclid begins his first book with five axioms. These are simple things that, he felt, needed no proof, like: "If equals are added to equals, the results are equal." He then lists five geometric properties of space that he calls postulates. The first four are very simple and needn't concern us; it is postulate five that has attracted attention throughout the centuries. It seems innocent enough when you first see it. We may state it as follows:

> *Suppose we are given a line and a point that is not on that line. Then there is one, and only one, line passing thru the given point that is parallel to the given line.*

Why all the fuss about so simple a statement? First, perhaps, because many people thought that it was something you should be able to prove; many tried, unsuccessfully, to do so. Secondly, it has an easy consequence that is rather striking. One can easily show, using this postulate, that the three interior angles in any triangle always add up to 180 degrees. The triangle can be long and thin or short and fat, as small as a button or as large as a galaxy; the three angles will always add up to 180 degrees.

Through the centuries various people tried to prove the fifth postulate from the other four, and some thought they had succeeded (Trudeau 1987: ch. 4). It was in the nineteenth century that two new geometries were created in each of which the fifth postulate was false. The first of these, hyperbolic geometry, was discovered independently, and pretty much simultaneously, by Nikolai Ivonovitch Lobachevsky and Janos Bolyai.

Here there is more than one parallel thru the given point. In the spherical geometry discussed by the great German mathematician Riemann, there are none. These were not immediately accepted as valid geometries until Eugenio Beltrami showed that the consistency of Euclidean geometry, which no one doubted, implied the consistency of the other two. This was a truly international effort; Lobachevsky was Russian, Bolyai was Hungarian, Riemann a German, and Beltrami an Italian (Trudeau 1987, Ramsay and Richtmyer 1995). Many more geometries are known today.

The equations of motion, those of Newton for example, are derived within a particular geometric model. The laws of motion can be formulated in any geometry, and the equations you get depend on the geometry you choose. Nature doesn't tell us which geometry we should use. This was best expressed by the great French mathematician Poincare:

> All measurement involves both physical and geometrical assumptions, and the two things, space and matter, are not given separately, but analyzed out of a common experience. Subject to the general condition that space is to be changeless and matter to move about in space, we can explain the same observed results in many different ways by making compensatory changes in the qualities we assign to space and the qualities we assign to matter. Hence, it seems theoretically impossible to decide by any experiment what are the qualities of one in distinction from the other. (Poincare 1952; see also Adler, Bazin, and Schiffer 1965: 1–16)

Humans see Euclidean geometry as very natural, almost obvious. In fact, the philosopher Immanuel Kant thought that this geometry was hard-wired into our brains and was the essence of how we see the world (Friedman 1992). It is so deeply embedded in our physics, astronomy, and higher mathematics that we are often unaware of it.

We cannot know, of course, what the geometric ideas of an alien society will be, nor can we know how they see the world, i.e., what geometric model will seem most natural to them. The laws of physics, however, are simplest when formulated in Euclidean geometry, and so is the associated trigonometry.

Even the fact that the circumference of a circle can be found by multiplying its radius by 2π, is false in these other geometries. The number π has no special significance in them (Ramsay and Richtmyer 1995: 13–14, 195). How often has this number been mentioned as something any intelligent race would certainly know?

An alien race that sees some other geometry as natural can develop a trigonometry and a physics, but these will be more complicated (McLeary 2002; and Lamphere 2002). Such complication may hinder progress. Unless they are super-intelligent, this may severely limit the progress they make in physics and even in astronomy. Could it be that the only races that make real progress in physics and astronomy are those that see the world around them as Euclidean?

A race that sees some other geometry as natural might still make considerable progress in chemistry and biology, and maybe those fields are the ones they will consider most important. This might seem very natural to beings who get their information mainly from their chemical senses and only make limited use of their sense of sight. It might be that a race will, in the course of its development, discover the Euclidean model and come to recognize the simplicity this model brings to the equations of motion and to the formulas of trigonometry. Such a race may choose to use this geometry at least for its scientific work, even if it is not entirely comfortable with it.

I should mention that there is a subject called "projective geometry" that arose from the study of perspective carried out by the artists of the Renaissance. Here, parallel lines meet as rail road tracks seem to do when you look down the track. But when scientists model the solar system, they invariably use Euclidean geometry.

Chapter 6

The Scotsman

Every successful space mission gets lots of media attention. This, in turn, invariably leads to letters to newspapers decrying the waste, or misdirection, of money and talent. Surely you've seen such letters. They often include some strange statistic and tend to be highly emotional: "Four hundred million dollars for a space probe!!! Why, for that amount of money, you could buy two hundred million goldfish; think about that!"

Such letters express a kind of tautology—something that is always true. No matter how you spend public money there are always those who say, sometimes with some justification, that it could have been spent on more pressing problems. Occasionally these letters say something about the mindset of those who write them. I remember one about the moon landing that read, "Why spend all that money just so some 'egg-head' can tell us that the moon is four billion years old?"

But one never really knows where a scientific investigation will lead or what the long range consequences of it might be. A dramatic case in point is the work of James Clerk Maxwell. His nineteenth-century highly theoretical investigations led to the development of radio, television, radar, and cell phones.

Now I am not suggesting that Maxwell is responsible for every idiot who insists on blabbing on his cell phone while driving at high speed through rush-hour traffic, or for every tasteless and embarrassing commercial that one sees on television (I once saw a deodorant commercial that featured a close up of Venus de Milo's arm pit!). That would be like blaming Columbus for every atrocity committed against the Native Americans. But Maxwell's work did make these devices possible and, in fact, made modern SETI possible.

In the nineteenth century there were many experiments conducted by scientists who were interested in understanding the interrelationships between electricity and magnetism. These experiments led to the concept of a magnetic field (Chapter 4). It was found that a charged particle also created a field, called an electric field, in the space surrounding it. The intricate relationships between these fields were investigated experimentally, and the results of these experiments were brilliantly summarized by Maxwell in a set of four equations that now bear his name (Zill and Cullen 2006: 486–88).

This sounds like something only other scientists would care about, another example of misdirected talent and money. Why wasn't Maxwell, who died in 1879, doing something useful? In those days when most people spoke of "something useful" they probably meant finding a better way to treat the hoof and mouth disease! Instead he spent considerable time investigating a curious consequence of his equations that, it seemed to him, had intriguing implications.

When you pluck a guitar string you set in motion a one dimensional wave. Beat on a drum and you produce a two dimensional wave and ocean waves are three dimensional.

In each case there is a mathematical description of the wave that is called, appropriately enough, the wave equation (Kreyszig 1999: 585, 616). Maxwell noticed that from his four equations, whose purpose, remember, was to summarize the inter-relationships between electricity and magnetism, he could derive the wave equation. What could this curious result mean? Perhaps it meant that there existed in nature an unknown, and totally unexpected, wave—an electro-magnetic wave? It was an interesting and provocative thought. He carried this research further, refined his thinking a little bit, and was able to calculate the speed of these hypothetical waves. The number he got was fantastic: 3×10^{10} centimeters per second, more than 186,000 miles per second! The scientific community was astonished, but also intrigued and impressed, because they had seen this number before. It is the speed of light.

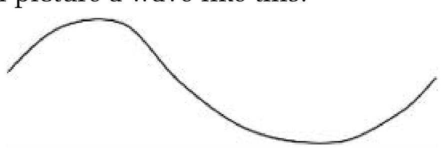
So the mathematical musings of a Scottish genius led to two unexpected and, at the time, totally useless conjectures: first, that there exists in nature a kind of radiation that humankind didn't know of and didn't even dream of; and, secondly, that light itself was nothing more than a form of this radiation.

Unfortunately Maxwell didn't live to see his twin conjectures verified and he surely never imagined the impact they would have on twenty-first-century life. It wasn't until 1888 that a German, Heinrich Hertz, first produced electromagnetic waves in the laboratory. The story goes that a student once asked what those waves were good for and, in reply, he shrugged and said, "Nothing I guess."

It was an Italian, Gugliemo Marconi, who, over a number of years, figured out how to use these waves to give us wireless communication. He was awarded the Noble Prize for these efforts in 1909. As already mentioned, the twentieth century saw these waves applied, by a great many talented and innovative people, to give us radio, television, radar, and, for better or for worse, cell phones.

But what does this have to do with SETI? That connection was not made until 1959.

We can picture a wave like this:



It starts on the left, rises to a maximum, then falls to a minimum and finally rises to its original level. This is called one "cycle," and the cycle is then repeated as the wave moves from left to right. The number of cycles that a wave goes through in one second is called the frequency of the wave. A frequency of one cycle per second is called one hertz (1 Hz) in honor of our German friend Heinrich. The AM radio stations typically broadcast at frequencies between 545 and 1605 kHz (one kilohertz equals one thousand hertz), garage door openers operate at about 40 MHz (one megahertz equals one million hertz), and baby monitors at about 49 MHz. FM stations broadcast at much higher frequencies, typically between 88 and 108 MHz.

These numbers may seem high, but they are below that of visible light. We, of course, perceive light of various colors. These correspond to the frequency of the light which varies from 4.3×10^8 MHz to 7.5×10^8 MHz, a very small portion of the electromagnetic spectrum (recall that 10^8 is a one followed by eight zeros). In order, from lowest to highest, we have red, orange, yellow, green, blue, indigo, and violet (just remember the name "Roy G. Biv"). Below red we have infrared radiation which we perceive as heat, and below that lie the radio waves. At the other end, above violet light, we have ultraviolet light, X-rays, and, finally, gamma radiation.

It wasn't until the mid twentieth century that people realized that many objects in outer space radiate not only in the visible light range but in the radio range as well. In 1931 a radio engineer, Karl G. Jansky, working for Bell Laboratories, tried to determine the source of high-frequency static that was disrupting trans-oceanic phone calls. After two years of work on the problem he was able to announce, at the meeting of the International Scientific Radio Union held on 27 April 1933, that the troublesome radio emissions came from outer space. This made the front page of *The New York Times,* but otherwise it attracted little attention. There was, however, one significant exception. A ham radio operator named Grote Reber built a 31-foot parabolic antenna in his back yard in Wheaton, Illinois, and thus became the world's first radio astronomer.

As the significance of this early work became better known, astronomers realized that they had a whole new window into the cosmos and the science of radio astronomy was born (Hey 1983). The field underwent explosive growth after World War II, and it must have occurred to many that we might accidentally hear the radio broadcasts of an alien civilization; in fact Marconi thought he had heard a signal from another civilization. Such speculation, however, was usually greeted with a tolerant smile, or a not-so-tolerant giggle, because even if such signals were out there, there was little chance that we would hit upon exactly the right frequency to enable us to detect them; unless there was a special frequency best suited for inter-stellar communication. Did such a "magic" frequency exist?

It was by suggesting a plausible answer to this question that two physicists ushered in the modern era of SETI.

REMARK: The Fundamental Wave Equation, Partial Differential Equations, Equations of Mathematical Physics, and the Function Concept

We have defined the frequency of a wave to be the number of cycles the wave goes through in one second. If we multiply this number by the length of the wave, we get the distance traveled by the wave in one second; i.e., we get the speed, or velocity, of the wave. This relationship is called the fundamental wave equation; it is not the equation that Maxwell derived—that one is much more sophisticated (see below).

Electromagnetic waves all travel at the same speed in empty space. This speed is usually denoted by the letter c, so $c = 3 \times 10^{10}$ centimeters per second. For these waves, then, we see that the higher the frequency is, the shorter the wavelength must be, because the product of frequency and wavelength must always be the same.

I would expect any society that can send electromagnetic waves across inter-stellar distances to be familiar with the principles encoded in the Maxwell equations. It is not clear that they will formulate these principles in the same way we do. The mathematics involved is called vector analysis and some of the concepts here are quite subtle.

It was known as early as 1747 that the motion of a vibrating string, like a guitar string, was governed by the one dimensional wave equation. This is an example of what mathematicians call a partial differential equation (see Chapter 9). Equations of this kind arise in problems of heat conduction, gravitational and electric potential, and many other problems of mathematical physics. Typically these equations have infinitely many solutions, and part of any problem involving them is to find the particular solution that satisfies an additional condition. In the case of the vibrating string we know the initial configuration of the string, and we seek a solution that, at the start of the motion, agrees with this configuration.

The discussion of the vibrating string, in particular, was the cause of a heated controversy that lasted more than ten years and had far reaching consequences for both mathematics and physics (see Remark, Chapter 7).

We have seen a number of correspondences between sets. Sometimes these were sets of numbers (Chapter 2), and sometimes these were sets of points in space and little arrows (Chapter 4). The general idea is this: We have two sets, say S and T, and we have a "rule" that sets up a correspondence between their elements. So to each element s of the set S, the rule must assign a unique element of T. We call such a rule a "function" (sometimes also called a "mapping" or a "transformation"), and we call S the domain of the function, and the set T the co-domain.

We are all very familiar with this idea. S could be the set of items in a store, and T could be the set of numbers, and the function could be the rule that assigns to each item its price. For each item we expect the price to be unique; we would certainly ask for clarification if an item had two different prices stamped on it. We would never confuse an object with its price.

It is important to note that a function need not be one-to-one. A function is one-to-one when any two different elements s_1 and s_2 in the domain are assigned to two different elements of the co-domain. The nice thing about functions that are one-to-one is that they have inverses; there is another function that "undoes" the given function (see Chapter 17). Most functions, however, are not like that. In any store, for example, we often have many items of the same price. Maybe a box of donuts costs the same as a can of shoe polish, but this doesn't confuse us—we certainly would never consider the two objects interchangeable. No one would ever tell his family that they'd be having shoe polish for breakfast because the store was out of donuts.

Functions are of fundamental importance in all areas of mathematics and they are important in many branches of science. When physicists speak of the distance a body travels over a period of time, this is a function. The velocity of that body is another function. When chemists speak of the atomic number of an element, this is a function.

A person's blood pressure and temperature are functions of time, and these can be of use to a physician.

The concept is everywhere in human science, and, as we have seen, shows up in the most ordinary aspects of life, like shopping. Is it possible to develop a science without recognizing this idea?

Chapter 7

The Birth of SETI

The extravagant claims of Percival Lowell about life on Mars caused many astronomers to shy away from the topic of extraterrestrial life. They did not flatly deny its existence but, for the first sixty years or so of the twentieth century, they rarely even mentioned aliens. But even in the 1960s we really didn't know much about conditions on the other planets of our solar system and couldn't rule out the possibility that advanced life existed somewhere in our neighborhood. One book suggested that the ubiquitous UFOs were from Mars and since these objects, according to witnesses' reports, were capable of accelerations no human could endure (the right angle turns reported, for example, would produce enormous accelerations), perhaps the pilots were intelligent insects. The body of an insect is inside its skeleton and hence they should be able to withstand greater G forces than any human could.

This imaginative writer specifically suggested that they might be bees (Heard 1951). Remember that many believed that Mars had plant life, so why not bees to pollinate all those plants?

We have already noted that Scully's book, "Behind the Flying Saucers" claimed that three saucers had crashed in the American southwest and that their occupants, all found dead, were small humanoids probably from Venus (see Chapter 4). But serious scientists dismissed such claims and were appalled by these lurid speculations; I think the idea of Martian bees riding around in spaceships really annoyed them. They might concede that there could be life on those two planets, but certainly not intelligent life. We were assured that the human race was alone in the solar system.

Strong human needs, however, don't simply go away and they are not easily stifled. Perhaps this is part of the reason that the mysterious UFOs were attracting more and more attention. Wouldn't it be great if one of them landed on the White House lawn? What would the know-it-all scientists say then?

But while the public was getting more interested in UFOs, and more people were willing to accept them as alien spacecraft, most scientists remained skeptical. They were convinced that we were the only intelligent race in the solar system and so, they reasoned, even if we do indeed have cosmic company it is located out among the stars. Interstellar travel is far beyond our capabilities (even today) and given the work of Einstein, it may be impossible. The stars are so far away that, until very recently, we couldn't even tell if they had planets around them; some believed that our solar system might very well be the only one in the galaxy. No, whatever the UFOs are, they are not spaceships, and the detection of an alien society, if one even exists, is something that we must leave to future generations. Since this was the prevailing attitude among scientists at that time, one can imagine the sensation caused by the paper "Searching for Interstellar Communications," published in the respected journal *Nature* on 19 September 1959. The authors were two prominent physicists, Giuseppe Cocconi and Phillip Morrison, both then at Cornell University.

They suggested that not only might intelligent alien life exist, but also we now had the means to detect it. Several parts of their paper are worth quoting to give insight into their thinking:

> No theories yet exist which enable a reliable estimate of the probabilities of (1) planet formation; (2) origin of

> life; (3) evolution of societies possessing advance scientific capabilities. In the absence of such theories, our environment suggests that stars of the main sequence with a lifetime of many billions of years can possess planets, that a small set of such planets, two (Earth and very probably Mars) support life, that life on one such planet includes a society recently capable of considerable scientific investigation. The lifetime of such societies is not known; but it seems unwarranted to deny that among such societies some might maintain themselves for times very long compared to the time of human history, perhaps for times comparable with geologic time. It follows then, that near some star rather like the sun there are civilizations with scientific interests and with technical possibilities much greater than those now available to us.
>
> To the beings of such a society, our sun must appear as a likely site for the evolution of a new society. It is highly probable that for a long time they will have been expecting the development of science near the sun. We shall assume that long ago they established a channel of communication that would one day become known to us, and that they look forward patiently to the answering signals from the sun which would make known to them that a new society has entered the community of intelligence. What sort of channel would it be?

This last sentence contains, of course, the crucial question, and Cocconi and Morrison spent a considerable time discussing it with each other and with colleagues before they finally hit upon an answer.

A typical atom consists of a nucleus orbited by negatively charged particles called electrons. The nucleus is made up of positively charged particles called protons and, in many cases, neutral particles called, as one might expect, neutrons. There are as many electrons as there are protons rendering the atom electrically neutral.

The simplest atom is that of hydrogen gas and this can be visualized as a single central proton orbited by a single electron. Every so often (the calculations say about once every eleven million years) the atom undergoes a "spin-flip" transition. When this happens the electron reverses its direction of rotation from clockwise to counter-clockwise or vice versa, and electromagnetic radiation is released. This occurs at a frequency of 1,420 MHz. As early as 1944 a Dutch scientist, Henk van de Hulst, suggested that there might be enough hydrogen in interstellar space for this radiation to be detected. If you have trillions and trillions of hydrogen atoms then, at any given time, some of them must be radiating. In 1951 he was proved correct when this radiation enabled astronomers to find enormous clouds of hydrogen gas. This discovery inaugurated a new era in radio astronomy (Verschuun 1974). But surely we can't be the only ones who have noticed this. Cocconi and Morrison reasoned that any scientifically advanced society would know of this "magic" frequency and would assume that it would also become known to any society as it matures scientifically.

Moreover, the electromagnetic spectrum is rather quiet in the region around 1,420 megahertz. This frequency then, is the unique way that distant societies could signal one another and begin a dialogue

Cocconi and Morrison ended their remarkable paper with the following provocative, and often quoted statement:

> The reader may seek to consign these speculations wholly to the domain of science fiction. We submit, rather, that the forgoing line of argument demonstrates that the presence of interstellar signals is entirely consistent with all we now know, and that if signals are present the means of detecting them is now at hand. Few will deny the profound importance, practical and philosophical, which the detection of interstellar communications would have. We therefore feel that a discriminating search for signals deserves a considerable effort. The probability of success is difficult to es-

> timate, but if we never search, the chance of success is zero.

Once this paper appeared the scientific community was faced with some tantalizing questions. Are the intelligent races in our galaxy in some kind of communication? Are they patiently waiting for us to "wake up and smell the coffee"? Do they share scientific knowledge? Is there some repository of cosmic wisdom that a race can tap into once it reaches a certain level of scientific maturity? Perhaps we can answer these questions by simply tuning our radio telescopes to 1,420 MHz? The observatories of the world, however, are busy places with time on various instruments assigned well in advance. Any new project must be carefully evaluated and, if accepted, then wait its turn. As luck would have it, that turn came sooner than anyone expected.

It has often been noted that ideas can occur independently, and more or less simultaneously, to more than one person. It was also in 1959 that a young astronomer named Frank Drake was at the newly built Radio Astronomy Observatory located in the mountains of West Virginia at a place called Green Bank. An 85-foot telescope had been built there, and Drake calculated that it could detect signals equal to the strongest signals we were then able to produce from a distance that included several sun-like stars (Blum 1990: 101). He wondered if, perhaps, one of them might be orbited by a populated planet. All it would take to listen for signals coming from these stars was to connect the right equipment to the telescope at hand.

He thought the matter over very carefully. After all, having his name associated with a search for alien intelligence could adversely affect his career. There was also the problem of buying the necessary equipment and getting permission to attach it to the telescope. Green Bank is a government facility and spending government money had to be justified.

One can easily imagine how the "bean counters" would react to hearing that tax payers' money was being spent to search for extraterrestrials: They'd act righteously indignant and gleefully try to portray Drake as a mad scientist who wanted to waste hard-earned public money on some crackpot idea; and would have gotten away with it were it not for their keen-sighted vigilance. Their shrieks and howls would probably frighten any nearby aliens into terrified silence.

But Drake saw a way around that particular problem. He calculated that, for about $2,000, he could build the equipment that, when attached to the telescope, would enable him to listen for radio signals coming from an alien civilization, if such a civilization was there and happened to be broadcasting, and at the same time, obtain information about the Zeeman Effect (a magnetic phenomenon). The latter topic was already under investigation at Green Bank and, by remarkable coincidence, the frequency at which it was being studied was 1,420 MHz. So he would be able to obtain useful information of then-current interest while pursuing his other, more daring, objective. By allowing the project the observatory would get two experiments for the price of one, and no one could claim that money was being wasted on "way out" ideas.

Drake was thinking politically as one must when working for the government. He presented his ideas to Lloyd Berkner, then director of the observatory, hoping that gentleman would see that the potential payoff of the project more than justified its modest cost. Berkner's reaction was all that Drake could have hoped for. He said, "Go ahead. Build it."

Drake was well into the construction of his equipment when the paper by Cocconi and Morrison appeared. He found it encouraging that, on the basis of scientific considerations, they suggested we listen at 1,420 MHz, the same frequency he had chosen for political reasons. One month later, the new director of the observatory, Otto Struve, announced during a lecture at MIT that Green Bank was preparing to listen for signals from outer space. The pressure was now squarely on Drake.

Finally, on 8 April 1960, he began what he whimsically called Project Ozma (named for a princess in the imaginary Land of Oz). He listened for 150 hours targeting two sun-like stars: Tau Ceti and Epsilon Eridani. There was a flurry of excitement when a pulsed signal was heard, but it was soon found to be of terrestrial origin; possibly a military aircraft (Blum 1990: 105–09). Unfortunately Project Ozma, and all subsequent searches to date, failed to detect an artificial signal.

Thus began the modern era of SETI, with a theoretical paper and a hands-on experiment. There was excitement, there was enthusiasm, and there was great hope.

But what was the real chance of success? And what could we hope to gain should we make contact? Could we really hope to talk to an alien race and, perhaps, learn from them, or should we fear the possible consequences of contact? The emotions that were prevalent in those heady days swept such questions aside.

REMARK: Two Functions and Why They Are Special, the Power of Trigonometry, and Fourier Series

Cocconi and Morrison suggested the frequency 1,420 MHz for inter-stellar communication. We can easily calculate the wavelength that a radio wave at that frequency must have because we know that its' velocity is $c = 3 \times 10^{10}$ centimeters per second (recall the fundamental wave equation stated in Chapter 6). The length of the wave turns out to be about 21 centimeters.

We have mentioned that astronomers first found distances in the solar system using trigonometry. The way this is done is to first set up functions from the set of angles, those between 0 and 90 degrees, and the set of real numbers. Here the fifth postulate of Euclid (Chapter 5) plays a fundamental role.

Since the interior angles of any triangle always add up to 180 degrees, a triangle can contain at most one 90 degree angle. When it does so it is called a right triangle and the side opposite the right angle is called the hypotenuse of the triangle.

Given an angle θ we find a right triangle containing this angle and associate with the angle two numbers. The first of these is called the sine of the angle, $\sin(\theta)$. It is the length of the side opposite θ divided by the length of the hypotenuse. The second is called the cosine of the angle, $\cos(\theta)$. It is the length of the side adjacent to θ divided by the length of the hypotenuse.

It might seem that these numbers depend not only on the angle but also on the triangle in which we have that angle. This is not so. Any right triangle that contains the angle θ must have all three angles equal, because the interior angles always add up to 180 degrees. That being the case, the triangles are similar which means the ratios mentioned above will be the same; there is a theorem of geometry that guarantees this.

This is what gives trigonometry its power. When an astronomer wants to find the length of the side of a triangle that is millions of miles long, he or she measures the relevant angle and makes use of the sine or cosine of that angle. To find that sine or cosine you don't need to use a triangle with million-mile sides. You can use a triangle that you draw on your note pad. As long as the angle is the same, the numbers you get will be the same. Of course, the sine and cosine of any angle needed can now be found on a hand calculator.

The sine and cosine are referred to as the trigonometric functions and they can be defined for any angle, not just those between 0 and 90 degrees. They are so simple and so useful that it is hard to imagine an alien race that is unfamiliar with them; how could they develop the technology to send radio waves across inter-stellar distances with out this knowledge?

Curiously, the trigonometric functions show up in problems of mathematical physics that have nothing to do with triangles. In the discussion of the vibrating string (Chapter 6), for example, they play a fundamental role.

In many problems, highly practical problems, of physics and engineering one has to solve a partial differential equation (Chapter 9) subject to an initial condition. This is often some known function that the solution must agree with at the start of the problem. In many cases this function must be expressible as an infinite sum of sines and cosines. This infinite sum—it is called a trigonometric series—must be given meaning in some way; this in itself is a non-trivial problem. There are many ways to assign meaning to an infinite sum.

An important kind of trigonometric series, now called Fourier series, were used by the French physicist Joseph Louis Fourier to solve a great many problems of mathematical physics. Fourier also stated that any function could be represented by such a series. It was in connection with this statement that people first began to realize that physicists and mathematicians live in different worlds. To a physicist a function is something "real," like the path of an object moving through space, or the initial configuration of a real string. The statement of Fourier makes sense in this context.

In the abstract world of the mathematician, however, lots of functions exist that have no immediate real-world interpretation, and for these the statement of Fourier is sometimes false. An embarrassing, and essentially fruitless, controversy, which lasted for about ten years, concerned solving the vibrating string problem using trigonometric series. The main point of contention was whether or not the initial configuration of the string could be represented by such a series. The argument centered on the meaning of the word "function," although those involved didn't seem to be aware of it (Jeffrey 1956: 5).

There is a subtle point here. If an angle is placed within a circle with its apex at the center of the circle it will intersect the circle in an arc. When the length of that arc is equal to the radius of the circle, we say the angle measures one radian. A 90-degree angle would have an arc equal to ¼ of the circle, or $2\pi r / 4 = (\pi/2)\ r$, where r is the radius of the circle. Thus 90 degrees corresponds to $\pi/2$ radians.

This is what enables us to use numerical variables in the sine and cosine function. But we should note that Euclidean geometry is behind this scheme. The circumference of a circle of radius r is $2\pi r$ only in this geometry.

Can we assume an alien race, even one with a sophisticated technology, will be familiar with these series? It seems rather brazen, perhaps arrogant, to say "yes," like saying our way is the only way. And yet the problems that lead to these series are real-world physical problems involving heat conduction in solids, wave motion, gravitational potential and electro-magnetic phenomena. Further more, these problems often can be shown to have unique solutions. So maybe assuming an alien society knows something about these series is not so unreasonable after all.

Chapter 8

The Conference at Green Bank

Right after Sputnik was launched, "space was the place"—it was where the action was and, with the remarkable achievements of the Soviet Union, the United States was faced with a new area of international competition. So the National Academy of Sciences organized a group of distinguished scientists charged with setting forth goals for an American space program that were both realistic and scientifically sound. This group, which came to be known as the academy's space sciences board, was chaired by Lloyd Berkner, the man who, as we have already seen, was to give the go-ahead for Project Ozma. The board members were well aware of how little we knew, in those early days, about what was "out there." Extraterrestrial life, even in our solar system, could not be ruled out. The possibility of such life had to be considered even though many believed it was the stuff of science fiction and not serious science. So a compromise was reached.

A member of the board's staff, J. P. T. Pearman, was asked to organize a conference to discuss, in the light of present day (1961) knowledge, the possibility of the existence of extraterrestrial societies, and the possibility of communicating with such societies. No public announcement was to be made however, and those invited were asked to keep the nature of the meeting as secret as possible. It was also suggested that the meeting be held in a remote location, away from the major academic centers and the big city news media. Berkner knew the perfect place: the radio observatory at Green Bank, West Virginia (Sullivan 1964; and Blum 1990: 110).

At that time the director of the observatory was Otto Struve (ironically Struve was a Russian émigré) and common courtesy dictated he should be the host and chairman of the conference. Struve agreed; his scientific work, as we shall see, also made him a good choice for this role. The actual planning of the meeting, however, he delegated to his young colleague Frank Drake.

Our sun is one of an estimated 400,000,000,000 (4×10^{11}) stars in the vast collection of stars, planets, gas, and dust called the Milky Way galaxy, a bared spiral, that, astronomers tell us, looks rather like a pinwheel and contains a central black hole (an exotic gravitational phenomenon that has worked its way into the public consciousness). An imaginative model of our galaxy was constructed by the artist Jon Lomberg on the big island of Hawaii. It is a garden that one can walk through and can be seen at www.galaxygarden.com. The Milky Way has a diameter of about 80,000 light-years and is about 2,000 light-years thick (Kaufmann 1994: 458). Our sun, which is located about 25,000 light-years from the center, rotates about that center at about half a million miles per hour. Still, it takes about 200 million (2×10^8) years to complete one orbit. Our galaxy is one of the two largest in a group of about 30 or so, called the local group. The other large member of this group is the great galaxy in Andromeda. There are an estimated 100,000,000,000 (10^{11}) galaxies in the visible universe.

These numbers, the number of stars in the Milky Way, and the number of galaxies have convinced many that life, even intelligent life, must have arisen elsewhere. This is not a proof, but a feeling based on the sheer magnitude of the universe and its age, which is estimated to be about

14,000,000,000 years. Life and intelligence did arise here, so why not elsewhere as well? But let us focus on our home, the Milky Way.

If there are other civilizations out there that are capable of interstellar communication, how can we estimate their number? This was the question Drake pondered as he prepared for the conference at Green Bank. He listed seven numbers that we needed to know that, when multiplied together, would give the number he wanted.

First we need the rate per year that stars form in the Milky Way. Next we need to know what fraction of those stars have planets. It is here that Otto Struve's work is relevant.

According to current ideas on stellar evolution, sun-like stars acquire a great deal of angular momentum during their formation—they spin fast. But Struve's study of a great many sun-like stars showed that they spin rather slowly. Angular momentum doesn't just disappear. It is an example of what physicists call a conserved quantity. So where did the angular momentum, acquired by these stars as they formed, go? In the case of the sun the answer is known: It was given to the planets. This, suggested Struve, might be where the angular momentum of the stars he studied went as well. In other words, these stars might have planets orbiting around them. Recent discoveries of extra-solar planets (several hundred have been found to date) tend to confirm this suggestion.

The next factor on Drake's list was this: Given that a star was orbited by planets, how many of those planets have environments suitable for life to arise? Here the work of another participant, the Chinese-American astronomer Su-Shu Huang, was relevant. He had studied the habitable zones around stars, the "Goldilocks" region where it was neither too hot nor too cold, so that any planet in orbit within that region might reasonably be the abode of life. Here is another way in which the Earth is special: It lies within the Goldilocks region of the Sun and so the water, so abundant on our planet, can remain liquid.

Next we need to know the fraction of those planets, those with environments suitable for life, on which life actually does arise; then the fraction of these on which intelligence emerges; and (almost finally) the fraction of intelligent societies that develop the ability and the desire to communicate with other worlds. If we knew these six numbers then, by multiplying them together, we'd have the rate per year at which such societies arise. At this point we assume a steady-state; i.e., the number of societies that die out is about equal to the number that begin to communicate, so that the number of such societies remains constant.

In order to get from a rate, so many societies per year, to the number of societies, we must multiply by a time. Drake called this seventh and final factor *L* and defined it to be the length of time that a given society remains technologically active.

Only the first of these numbers, the rate per year at which stars form in the Milky Way, is known with any certainty even today. The last factor, *L,* was especially troublesome back in 1961. At that time, deep in the Cold War, the United States and the Soviet Union each had the capability to annihilate the other and, in so doing, unleash a "nuclear winter" that might very well destroy the human race. Many wondered if, perhaps, when a society finally reaches the point where it is capable of interstellar communication, it then gleefully proceeds to destroy itself. This would certainly explain the Fermi paradox.

As I have said, six of the seven factors listed above are still unknown. Drake gave each a letter designation and set their product equal to *N,* the number of communicating societies in the galaxy. This is now called the Drake equation (see the Remark below). It is a way of organizing our thoughts about extraterrestrial societies, some say a way of organizing our ignorance about such societies. It was never meant to be a precise formula for *N,* but rather a way of roughly estimating this number. This is a reflection of the fact that, in this area, our knowledge is meager and uncertain.

But let's return to the Green Bank conference. Who else was there? In addition to Pearman, Drake, Huang, and Struve, the conference included Dana Atchley, a communication specialist; Melvin Calvin, a chemist (more about him later); Guiseppi Cocconi and Philip Morrison, the men who wrote the seminal paper; John Lily, a dolphin researcher; Bernard Oliver, then vice president of research at Hewlett-Packard; and Carl Sagan. Many will remember Sagan as the host of the popular television program *Cosmos* and the author of the novel *Contact* (a movie of the same name was based on this novel). These dealt specifically with SETI, and Sagan, a charismatic and articulate planetary scientist, was the best-known and most influential advocate of this endeavor until his untimely death in 1996.

A provocative talk, some say a challenge (Blum 1990: 110–11), about inter-species communication was given by John Lily, who then headed the Communication Research Institute. He was convinced that dolphins are intelligent and that they have a language. At that time, researchers at the Institute, which was located in the Virgin Islands, were investigating the possibility of man-dolphin communication.

It must have seemed like a good "omen" when, while the conference was in progress, Melvin Calvin was awarded the Nobel Prize for Chemistry (on the basis of his groundbreaking research on photosynthesis). Anticipating this, Pearman (obviously a very wise man) had brought along a number of bottles of champagne. The popping of these corks must have been a high point in the meeting. Perhaps it was Lily's talk, together with the champagne, that led the group to refer to themselves as the "Order of the Dolphin." Afterwards, Calvin had pins made up showing a leaping dolphin and sent one to each participant.

The conference at Green Bank was an important event in the development of SETI. It generated much excitement and enthusiasm among its participants, and this is always of importance in any undertaking. Furthermore, the fact that all these distinguished scientists had gathered to seriously discuss SETI gave the subject a new respectability; the ridicule barrier had been broken or, at least, dented. The question of how we might go about communicating with an alien society seems to have received little attention; there is only so much you can do at any one conference. A book dealing with that very question, however, had been published in 1960 by the respected Dutch mathematician, Hans Freudenthal (more about this in Chapter 11).

REMARK: The Drake Equation, Drake's Postcard, and Prime Numbers

The Drake equation is usually written as follows:

$$N = Rf_p n_e f_l f_i f_c L$$

Here R is the rate at which stars are formed in the galaxy per year; f_p is the fraction of stars that have planets; n_e is the number of these planets that have environments suitable for life to arise; f_l is the fraction of these planets where life actually evolves; f_i is the fraction of f_l on which intelligent life arises; f_c is the fraction of f_i where a culture capable of communicating over interstellar distances arises; and L is the average time that such a society remains technologically active. Only R is known with any certainty even today.

A probabilistic version of the Drake equation has recently been given by Claudio Maccone (Maccone 2012). It is a remarkable work and one that I think, as it becomes more widely known, will have far reaching consequences.

As I stated above, the participants in the conference at Green Bank referred to themselves as the "Order of the Dolphin." This was, perhaps, due to the influence of John Lily. I should mention that another dolphin researcher, Denise Herzing, is active in SETI. She often speaks at professional conferences using dolphins as an example of non-human intelligence. Her fascinating book (Herzing 2011) describes her work with these amazing animals.

It was shortly after the conference that Drake came up with an idea that could be used as a kind of "hailing message" to any society out there. Messages based on this idea have been sent into space. The thinking is that any scientifically sophisticated society would know something about the natural numbers, something beyond just using them for counting. In the movie *Contact* the aliens call attention to their signal by sending a series of prime numbers (these are numbers, larger than 1, whose only factors are themselves and 1, like 2, 3, 5, 7, 11, 13, . . .). Drake's idea is based on the fact that any whole number larger than one is either a prime or it can be factored into primes. The number 30, for example, is the product of the three primes 2, 3, 5, and the interesting thing is that these are the only primes whose product is 30. So anyone, or any thing, who tries to break 30 into primes will get the same factors.

Given a natural number n, we look at all numbers less than n, except the number 1, and see if any of them divides evenly into n. If none do, n is a prime. Otherwise, the first (smallest) number that divides n, call it p_1, must be a prime, because any number that divides p_1 must also divide n, and p_1 is the *smallest* such number, so p_1 can't have any divisors other than itself.

Now $n = p_1m_1$, for some natural number m_1. So we look at m_1. If m_1 is a prime then n is the product of two primes. If m_1 is not a prime it has a smallest divisor p_2, and p_2 is a prime. So $n = p_1p_2m_2$ for some natural number m_2. Continue in this way. The process comes to an end when n is the product of only primes.

Our discussion shows that any number greater than one can be written as a product of primes. The more difficult part of the theorem is showing that the primes you get are unique. This requires some number theory.

Drake's message, which he presented to his colleagues as a transmission form a hypothetical alien civilization, consisted of 1,271 zeros and ones. What is one to make of such a sequence? Since we can't think of anything else to do, let's factor 1,271 into primes. The number 1,271 is the product of two primes, 31 and 41. Now this is a message from another world, so we take every aspect of it seriously. The first question then is why are there only two prime factors? Presumably the aliens would deliberately choose a number with only two prime factors for a reason. Maybe it is to tell us that the message is two dimensional; i. e., maybe it is a picture.

Arranging the zeros and ones into 31 rows and 41 columns gives a meaningless array (at least we can see that it is meaningless), while arranging these symbols into 41 rows and 31 columns yields a remarkably detailed picture—a picture packed with information about our correspondents. It tells us something about their "solar system", their biology and their mathematics. Messages based on this idea have been sent into space. Whether or not they would mean anything to an alien race is, of course, anybody's guess.

There is no end to the primes. No matter how far out in the sequence of natural numbers you go, there are always primes farther out. There is no formula giving all the primes, although some come tantalizingly close. One can show that when n is *not* a prime, neither is 2^n-1 (DeVito 2007: 159). When n is a prime this number gets more interesting. Let's run through the first few primes, 2, 3, 5, and 7. The number 2^2–1 is the prime 3, and 2^3–1 is the prime 7. Also 2^5–1 is the prime 31, and 2^7–1 is the prime 127. Things are looking good!

But the next prime is 11, and the number 2^{11}–1= 2,047 is not a prime; 2,047 is 23 times 89.

It has been said that Coronado never did find the seven cities of Cibola, but he did find a place in which to look for them. In a way the sequence 2^p–1 is like that. It doesn't always give us primes, but it does give us a "place" in which to look for them.

The numbers of the form 2^p–1 were discussed by the Frenchman Mersenne, and even earlier by Euclid. They are now called Mersenne numbers, and there is a test, the Lucas-Lermer test, that tells us when one of these numbers is prime. Lucas showed, for example, that the number

$$2^{127}-1 = 170{,}141{,}183{,}460{,}469{,}231{,}731{,}687{,}303{,}715{,}884{,}105{,}727$$

is a prime. No, it is not the largest one known, but it certainly is pretty big (DeVito 2007: 162). Large primes play a role in securing data, and in ciphers.

Since the primes are the "elements" of the natural number system, they may be known to any society that has a science. They seem to be a good way to call attention to a message and make clear that the message is the product of intelligence, and not just a natural phenomenon.

Chapter 9

Stories—Part Two

If you ever give a public talk about SETI you will find that, as likely as not, someone will ask you about the Roswell incident. When this first happened to me I had to admit that I didn't know anything about it. So, I did some reading, and this is what I learned.

In July of 1947, just a few weeks after Kenneth Arnold made his famous sighting, a UFO allegedly crashed on a remote ranch near Corona, New Mexico. Subsequent events have forever linked this incident to the town of Roswell, about seventy-five miles to the southeast and the nearest town of any size (Berliner and Friedman 2004). Roswell Army Air Field was the home of the elite 509th Bomb Group charged with handling and, if necessary, delivering America's atomic bombs. It was commanded, at this time, by Colonel William "Butch" Blanchard. What happened there that fateful summer, and even exactly when it happened, is a little difficult to pin down. There are now at least six versions of the story (Saler, Ziegler and Moore 1997: 17–29) and the claims and counter-claims found in the many books about the incident can be bewildering (see the first six references below). There seem to have been three incidents, perhaps related perhaps not, that have come to be combined into one story. Moreover the social, or historical, ideas current when the story first came to light might be significant.

After the incidents described in Scully's book had been discredited, back in 1952 (Chapter 4), those interested in

UFOs were generally skeptical, and extremely wary, of all stories involving crashed saucers. This attitude, however, was to change, radically, in the late 1970s. In a book titled *Situation Red, The UFO Siege* (Stringfield 1997), and in a lecture given at a UFO convention in July of 1978, Leonard H. Stringfield told of nineteen such crashes; he called them "retrievals of the third kind." These were based, he said, on the testimony of twenty-two military men and civilian professionals who had taken part in the retrievals. These stories were endorsed, the next year, in *Flying Saucer Review.* Not only were they endorsed, but Scully's book was also completely reevaluated and, in that same issue, Editor Gordon Creighton, wrote:

> The Scully book was dynamite, and it naturally created a sensation. It was therefore imperative that Scully be stopped in his tracks, and a feverish and powerful campaign was at once launched to damn and discredit him utterly. That campaign was 100 percent successful.

Crashed saucer stories, it seemed, were once again acceptable and, furthermore, they became difficult to falsify: If they weren't discredited then they must be true, and if they were discredited, then it was because the government (or some portion of the government) didn't want you to know that they were true (Saler, Ziegler, and Moore 1997; see also Berliner and Friedman 2004: 41–46). It was in this atmosphere of acceptance of crashed saucer stories and suspicion of those who tried to discredit them, that the Roswell story was rediscovered.

The first of three incidents making up the story is the sighting of a saucer-like object over the town of Roswell on the evening of 2 July 1947. The witnesses, Mr. and Mrs. Wilmot, did not report the sighting, however, until 8 July. Their report was, in fact, part of the same article in the local newspaper, the *Roswell Daily Record,* that contained the announcement that the wreckage from a flying saucer had been recovered by the military (see Chapter 4).

The second incident is much more complicated. It concerns the debris found on the Foster ranch by the ranch manager William "Mac" Brazel. A violent thunderstorm had passed over the area on the night of 2–3 July and, the next morning, Mac and a neighbor were out on horseback checking the range for damage to fences and windmills. Several miles from the ranch house they came upon a strange sight. Scattered over a wide area about three quarters of a mile long and several hundred feet wide (Berliner and Friedman 2004: 99) was what appeared to be some kind of wreckage; neither rider had ever seen anything like it before and Brazel, ever the practical cowhand, wondered how he was going to clean it up. He became more concerned with this question when he found that his sheep wouldn't go across it even though their water source was on the other side (Marcel 2007: 24). They had to be led around the area.

The debris that Brazel and his neighbor found consisted of small pieces of foil-like material, strong thread-like material (it didn't have fibers as real thread does) and pieces of wood-like material that was light but very hard. The foil, if it was wadded up and placed on the ground, would spontaneously unfold itself. Also, some of the material had "figures," described as resembling hieroglyphics, printed or embossed on them in a pink or purplish ink.

It has been suggested that the flying saucer seen by Mr. and Mrs. Wilmot ran into the storm over the Foster ranch and was damaged by the wind or the lightening. This, it has been said, was the source of the material Mac Brazel found. But no one who saw the debris ever mentioned seeing anything that resembled a seat or an engine or any evidence of a crew. So if all this did come from a flying saucer, where was the rest of it? This is where the third incident comes in.

This is the most intriguing incident of all. In 1978 Vern Maltais told a UFO investigator that a close friend of his, an engineer named Grady "Barney" L. Barnett, was in

New Mexico in 1947. According to Maltais, Barnett (who was deceased) said that he had come upon the wreckage of a flying saucer and the bodies of its crew. A group of archaeology students from the University of Pennsylvania were also at the site. This was on 3 July 1947, on the Plains of San Agustin, about 150 miles from Roswell. Remarkably, military personnel arrived almost immediately. They chased everyone from the site, took all of their names, and told them it was their patriotic duty to tell no one what they had seen (Berliner and Friedman 2004: 87–88). Was this perhaps the craft that Mr. and Mrs. Wilmot had seen and the source of the debris found by Mac Brazel? Some people think so (Berliner and Friedman 2004).

These three incidents seem to be at the heart of the Roswell story. They are linked, however, only by the dates on which they occurred, and therein lies a serious problem. In an article published on 9 July in the *Roswell Daily Record,* Brazel said that he found the debris on 14 June, not 3 July (Berliner and Friedman 2004). If this is in fact the correct date, then linking the three incidents becomes highly questionable. There are other problems with the story as well. Barnett's wife kept a diary and, according to her, Barney was in a different part of New Mexico, nowhere near the Plains, on 3 July. Furthermore, none of the archaeology students has been located. You'd think that, with all the media attention this incident has received some of them would have come forward (Peebles 1994: 248).

It isn't just the saucer sighting by Mr. and Mrs. Wilmot that links this story to Roswell. On the evening of 5 July, Brazel went to the little town of Corona and was told, by his uncle Hollis Wilson, about the flying saucer craze that was then current. Perhaps, he suggested, the material Mac had found came from one of these? Mac had planned to drive down to Roswell anyway, a long and arduous trip over unpaved roads, and so, on 6 July, he made the trip and took along some of the debris. This he showed to George Wilcox, sheriff of Chaves County, and he, in turn, called the Air Base. The end result was that the base intelligence officer, Major Jesse Marcel, drove out to the ranch along with counter-intelligence officer Captain Sheridan Cavitt.

The men went in three cars and arrived at night, too late to do much of anything. The next morning they went to the debris field and loaded what they could into the two officers' cars (Berliner and Friedman 2004: 82). They spent most of the day collecting material, and a great deal still remained on the ground when they left.

Back at the base Marcel stopped at his home, woke up his wife and their 11-year-old son, and showed them the material he had collected. He believed that he had the wreckage of a craft from another world and he wanted them to see it before the whole incident became "classified" (Berliner and Friedman 2004: 74). The next day, under orders from Col. Blanchard, Lt. Walter Haute issued the famous press release (Chapter 4).

It is curious that Barney Barnett said that on 3 July, before Brazel even got to Roswell, the military arrived at the crash site soon after he did. How did they know about the crash? That objection has been discussed. New Mexico was the home of many of the nation's most secret military installations and many of these were protected by radar. It has been suggested that an object was sighted on radar and its crash noted but not pinpointed. At first light military personnel were sent out to find the wreckage. At this time they expected only to find a downed airplane. When they came upon a saucer and alien bodies they immediately did what they could to contain the story (Randle and Schmitt 1994).

Some say that two UFOs crashed that summer. Perhaps, due to the intense rain and lightening, they collided in midair. The first one found was on the Plains of San

Agustin and the military immediately took charge of it. When Marcel returned to the base with the material he had collected at the Foster ranch, however, the military commander realized that there might be a second saucer out there. A search was then conducted and the main portion of the UFO, and the bodies of its crew, were located several miles from the debris field. The craft and the bodies were then taken to Roswell Army Air Field.

The Roswell story has become an integral part of modern UFO-lore. But it suffers from the same weakness that most UFO stories do. It is based on the testimony of "people whose word would be accepted in a court of law." The phrase in quotes is commonly used by UFO believers who seem to think it proves something. But, as anyone knows, people who testify in court are not automatically believed. They are questioned closely by lawyers and, in many trials, physical evidence, the testimony of experts, and the testimony of other witnesses are brought in to confirm or contradict what they say.

And let's not forget that sane, sober, respectable people have claimed that they saw ghosts, women riding broomsticks, Bigfoot, tiny humanoids with wings, the Loch Ness monster, and Elvis Presley after he died. You can bet that some of these sightings are hard to explain. Are we to believe all of this? If not, where does one draw the line?

Furthermore, when one UFO incident is explained the believers immediately come at you with more stories. After so many years and so many sightings you'd think that a good, clear picture, preferably a motion picture showing real detail, would be widely available. There have been thousands, perhaps tens of thousands, of reports. Cameras, of one sort or another, are ubiquitous. Yet no clear, non-controversial, detailed image of a UFO has been captured. Moreover, with all the UFOs flying around for so many years you'd think that some would have crashed due to equipment failure, pilot error, or just bad luck. Apparently it happened once, and only once, and that was at Roswell.

The only physical evidence that this crash happened is the debris that Mac Brazel found on the ranch. The Air Force has an explanation for that. They now admit that the weather balloon story was a cover-up (Chapter 4). During the first week of June, however, three balloon *trains* were launched from Alamagordo Army Air Field. These were part of what was then a top secret project called Mogul, designed to detect Soviet nuclear tests. Each train contained about two dozen neoprene balloons. One of the people involved in the project, Charles B. Moore, claims that they used a scotch-like tape in constructing these balloon trains and furthermore, that the tape had flower-like designs printed on it with a pinkish-purple ink. This, the Air Force now claims, is what Brazel found on that morning. These balloon trains were launched in early June so the wreckage was probably found on 14 June, not 3 July. Needless to say, not everyone is happy with this explanation (McAndrews 1997). I once mentioned it to an acquaintance and got this reaction: "I'd rather believe it was aliens." What can you say to that?

There is one other story of a crashed saucer that has received some scientific attention. This involves three pieces of metal that allegedly came from a flying saucer that crashed in Brazil. These were tested by several metallurgical experts with various, sometimes contradictory results; these are discussed by Story and Greenwell (1981: 100–07). The story begins with a letter addressed to Ibrahim Sued, a columnist for the Rio de Janeiro newspaper *O Globo*:

> Dear Mr. Ibrahim Sued.
>
> As a faithful reader of your column, and an admirer of yours, I wish to give you something of the highest interest to a newspaperman, concerning the flying saucers. If

> you believe they are real, of course. I also didn't believe anything said or published about them. But just a few days ago I had to change my mind. I was fishing together with some friends at a place near the town of Ubatuba, Sao Paulo, when I saw a flying disk. It approached the beach at unbelievable speed, an accident seeming imminent—in other words, a crash into the sea. At the last moment, however, when it was about to strike the water, it made a sharp turn upwards and climbed up rapidly in a fantastic maneuver. We followed the spectacle with our eyes, startled, when we saw the disk explode in flames. It disintegrated into thousands of fiery fragments, which fell sparkling with magnificent brightness. They looked like fireworks, in spite of the time of the accident—at noon. Most of these fragments, almost all, fell into the sea. But a number of small pieces fell close to the beach and we picked up a large amount of this material—which was as light as paper. I enclose herewith a small sample of it, I don't know anyone that could be trusted to whom I might send it for analysis. I never read about a flying saucer having been found, or about fragments or parts of a saucer that had been picked up; unless it had been done by military authorities and the whole thing kept as a top-secret subject. I am certain that the matter will be of great interest to the brilliant columnist and I am sending two copies of this letter—to the newspaper and to your home.

The letter contained three pieces of metal. One can not help but suspect a hoax here. Why was the letter sent to Mr. Sued? He was the society editor of the paper. What interest would he have in flying saucers? Why was the signature on the letter illegible and why was no return address given? There is further evidence that casts doubt on the incident described in the letter. The Brazilian representative for APRO (aerial phenomena research organization, a private group with headquarters in Tucson, Arizona) was, at this time, Dr. Olavo T. Fontes. He conducted an extensive investigation to try to locate any witness to this extraordinary event. His efforts were unsuccessful. Not one single witness was found.

Mr. Sued did give Dr. Fontes the three metal pieces that were contained in the letter and he, in turn, had one of these analyzed by the Mineral Production Laboratory, a division of the Brazilian government's National Department of Mineral Production. They found that the material was very pure magnesium. In the process of testing, however, the sample was destroyed.

The other two samples were sent to APRO headquarters. One of these was analyzed by Dr. Roy Craig, a member of the Condon committee (see Chapter 16). Dr. Craig found that the material was not as pure as the Brazilian laboratory had stated and that, in fact, experimental batches of magnesium were produced by the Dow Metallurgical Laboratory that were of similar purity as early as 1940. He said (Story 1981, 102): "The claim of unusual purity of the magnesium fragments has been disproved. The fragments do not show unique or unearthly composition, and therefore they cannot be used as valid evidence of the extraterrestrial origin of a vehicle of which they are claimed to have been a part."

The matter did not end there however. In 1969 APRO gave another sample to Dr. Walter W. Walker, a professor of metallurgical engineering at the University of Arizona. He found, after non-destructive testing of the sample, that the fragment was a directionally solidified casting (this is a process whereby the molten metal solidifies in such a way that the metallic crystals are all in one direction). He commented, "This might be interpreted as meaning that the samples were from a more advanced culture."

Yet another analysis was conducted by Dr. Robert E. Ogilvie, a professor of metallurgy at the Massachusetts Institute of Technology. He said:

> Results of these tests showed the metal to be pure magnesium. No impurities or alloying elements, such as

> aluminum, zinc, manganese, or tin, were found. An oxygen x-ray map picked up magnesium and oxygen x-ray signals, thus confirming the network [of a white powdery substance in the surface cracks] to be magnesium oxide.
>
> My conclusion is that the specimen from Brazil has a composition that would be found in magnesium weld metal. However, the structure is indeed unusual. In my opinion it could only have been formed by heating the magnesium very close to its melting point in air. It would be necessary to hold the temperature for only a minute or so. This would produce an oxide coating on the material, which is clearly visible. Also, oxygen would diffuse down the grain boundaries, thereby producing the oxide network. It is therefore quite possible that the specimen from Brazil was a piece of weld metal from an exploding aircraft or a reentering satellite.

In 1980 Dr. Walker was asked to comment on the various investigations of the material. He said:

> The original analysis by the Brazilian Mineral Production Laboratory indicated no impurities in the magnesium which is still a terrestrial impossibility. Unfortunately, this entire sample was destroyed, so verification of its singular purity is impossible. However, the method used should have involved photographing the emission spectra on film. The film may still exist, even though the sample was destroyed. If that film is still in existence, it might serve as evidence of the possible extraterrestrial origin of the now destroyed Ubatuba No. 1.
>
> Subsequent analysis of Ubatuba No. 2 and No. 3 have all indicated a normal total-impurity level. All these later analyses have proved is that Ubatuba No. 2 and No. 3 were possibly from a different source than Ubatuba No. 1. The implication in Craig's statement in the Condon Report is that Ubatuba No. 2, which he analyzed, came from the same source (casting, weld, or what ever) as Ubatuba No. 1. We have absolutely no proof

that this is so. All we do know (if we accept the original story) is that all three pieces are from the same vehicle, but we have no reason to believe that they are necessarily from the same part of that vehicle.

As to my metallurgical finding that the two small surviving pieces examined were directionally solidified: All welds and castings have small areas which exhibit directional solidification, and the Ubatuba fragments are so small that we cannot conclude the entire weld or casting was directionally solidified.

The important thing about directionally solidified, commercially pure magnesium is that it is too weak to use structurally, even on subsonic, terrestrial vehicles. The yield strength is less than 10,000 PSI. Magnesium alloys are widely used, since adding substantial quantities of aluminum, zinc, thorium, rare earths, etc., imparts strength to the basically weak magnesium and raises the yield strength to usable levels. To my knowledge, no commercially pure magnesium is used in any vehicle, either as castings or weld-filler metal. Hence, Ogilvie's suggestion that the Ubatuba sample was an unalloyed magnesium weld, and therefore from an exploding terrestrial aircraft or satellite, is unacceptable.

Ogilvie falls into the same trap that many critics of the Ubatuba incident fall into. They all assume that commercially pure magnesium is used as either weld-filler metal or as castings on flight vehicles. Therefore, the source must be an exploding aircraft, rocket, or satellite. Since commercially pure magnesium is not used for either castings or welds in flight vehicles, such explanations are weak. All such critics should look more at the rather peculiar origin of these samples and less at exploding aircraft, rockets, or satellites.

The major use of commercially pure cast magnesium is in the form of cast, sacrificial anodes for corrosion control. [Note: A sacrificial anode is a piece of metal that is allowed to corrode so as to protect another metal].

Anodic protection involving these anodes was practiced in all developed countries, including Brazil,

> in 1957. Because of the casting method used, these anodes display zones of columnar, directionally solidified grains. The scenario is, then, that sometime in 1957, a metallurgically sophisticated person or persons secured three small anode pieces and sent them to Ibrahim Sued, with the dramatic description of the event at Ubatuba.

It seems pretty clear, after examining all the evidence presented here, that the Ubatuba sample is not part of a UFO but merely a rather cleverly presented hoax. This incident also shows, rather forcefully, why scientists are reluctant to get involved in any investigation of UFOs. After all the time and effort spent by the Brazilian laboratory and three separate scientists, what has been accomplished?

REMARK: Development of Calculus, Models for Time, Differential Calculus and the Science of Motion, and Derivatives and Partial Derivatives

Roswell is a story about space travelers. Such travelers, whether human or alien, must have some understanding of the properties of motion. This is something the ancient Greeks had a great deal of trouble with as the paradoxes of Zeno show. At the human level motion is a continuous process. Understanding this process required that we refine our conception of the continuous and learn how a discrete process, repeated infinitely often, can capture the essence of a continuous change. This aspect of human perception arises again and again in mathematics and we shall have occasion to discuss it further later on (Chapters 10 and 11). The development of calculus is essentially a story of how we made our intuitive understanding of space, time, and motion mathematically precise.

The scientific study of motion began with some observations and experiments by Galileo. He was the first to realize that a material body has “inertia”; it resists changes to its state of motion. The mass of a body is a measure of this resistance. In ordinary language the mass of a body is regarded to be the same as its weight. A body in space, however, still has mass, still resists changes to its state of motion, even though it is weightless. Newton took this as his first law of motion and stated it as follows:

> *A body either remains at rest or continues moving at constant velocity in a straight line unless acted on by an external force.*

Newton's second law states that a force acting on a body produces an acceleration. The magnitude of the acceleration is proportional to the strength of the force. If you double the force, you double the acceleration, triple the force and you triple the acceleration, and so on. This means that the force is a constant times the acceleration. The constant of proportionality according to Newton, is equal to the mass of the body. So the force is equal to the acceleration times the mass.

In order to get a deeper understanding of this law we must say something about time. In the year 470 the Roman philosopher Martianus Capella suggested that there might be "atoms" of time (Whitrow 1961). There is some evidence that this view was very popular in the past. In the Italian language, for example, the word for instant, *attimo*, is pronounced very much like the word for atom. We also know that the concept was discussed by Islamic scholars, but the first person to assign duration to an atom of time was Rabbi Maimonides. In his book *The Guide for the Perplexed,* written in the twelfth century, he suggested that there might be 60^{10} atoms of time in an hour. His reasoning may have been something like this. There are, of course, 60 minutes in an hour, 60 seconds in a minute, perhaps 60 sub-seconds in a second, and so on. At the tenth step we reach the atoms of time which can not be divided further. Why stop at ten? I don't know what the Rabbi was thinking, but it makes sense to me. Most of us have ten fingers, and how could that not be important?

By the time of Galileo, however, the idea of atoms of time was replaced by the current idea that time is a continuum. It consists of duration less instants (for how long does the clock say twelve?), and any segment of time can be divided into arbitrarily small sub-segments. The importance of this is that in the equations of physics the velocities and accelerations must be instantaneous velocities and instantaneous accelerations. If we travel from Tucson to Phoenix in three hours, then our average velocity is 120 miles divided by 3 hours, or 40 miles per hour. But the car's speedometer doesn't say 40 throughout the trip. It tells us the speed at a particular instant. To make this precise requires the ideas from calculus. In fact, one of the fundamental problems of differential calculus is to find, given the distance traveled by an object as a function of time, the velocity of the object at a specific instant.

Differential calculus is an extremely useful subject. It is fundamental in the science of mechanics and in other areas of physics and engineering. But would an alien race know this subject, or would they have some other way of dealing with the subtleties of motion?

It is interesting to note that many people find the basic ideas of calculus intuitively pleasing, almost obvious. These are usually people who have good physical insight, good intuition for the properties of motion. This may go back to our ancient ancestors. When fleeing a predator you can't think about how to run or how to jump for a tree branch, you have to react and do it quickly. So those who survived are those with a good sense of motion. Even when we were the predator we needed to understand motion. You have to know, without thinking, where to aim your spear at a fleeing animal in order to hit it. This kind of "intuitive" understanding is enough for those who want to use calculus as a tool. A deeper understanding of the subject requires more abstract reasoning.

Will an alien race share this with us? If they have a very different evolutionary history they may never have internalized the properties of motion. This may have an effect on the kind of mathematics they develop. Perhaps calculus will never occur to them or, if it does, it may be considered an esoteric subject of little practical importance. It is very hard to imagine a science without this subject. It is certainly something we would look for in any race we contact.

Years of experience impels me to point out that calculus is not the end of mathematics. This belief is common among students but also among highly educated people. Every scientist knows that the literature in physics is extensive and constantly growing, and it is impossible for any one person to know it all. Surprisingly, few seem to realize that this is true of mathematics also, and has been so for many years.

Calculus is based on some very sophisticated ideas, and it took many decades to properly lay the foundations of the subject. This is what distinguishes it from the more elementary branches of mathematics. We must give precise meaning to quantities that involve infinite processes like the instantaneous velocity of an object, which is done by taking average velocities over smaller and smaller intervals of time. The calculation is called "differentiation" and it is one of the two fundamental operations of calculus.

If we have the distance an object travels as a function of time, then the velocity of the object (at a fixed instant) is the "derivative" of the distance with respect to time. The derivative of the velocity with respect to time is the acceleration of the object. This is the "second derivative" of the distance.

An equation involving derivatives is called a differential equation. Many of the laws of physics are stated as differential equations. Solving these equations requires that we "undo" the differentiation. This process, called "integration," is the second fundamental operation of calculus (see Chapter 10).

When a function depends on more than one variable we can fix all but one of these and carry out the process of differentiation as if the function depended only on the one variable that is not fixed. When we do this we get the "partial derivative" of the function. An equation involving partial derivatives is called, not surprisingly, a partial differential equation. As we have already noted (Chapter 6) such equations play an important role in physics and engineering.

Chapter 10

Talking to E.T.

As we have already noted any alien society will live out among the stars. The enormous distance between us will make communication technically challenging, but it will also make inter-stellar aggression highly unlikely. The expense and the energy required to launch an expedition across these vast gulfs would far exceed any possible gain. But should we even try to contact such a society? What could we hope to gain? Does it make any sense to engage in a dialogue where the interval between a message and its response is measured in years? These questions will be the subject of the next few chapters. As usual, it is a lot easier to ask them than it is to answer them. Each one requires an extended discussion in order to bring all of its ramifications into focus (see the first three references below). Let us start here with what is, perhaps, the most basic one. Is communication between aliens possible?

It has been said, and it is often repeated, that "Even if a lion could speak English, I would be unable to understand him." The point being made is, of course, that the world of the lion, his concerns, his interaction with his pride, even his conception of his surroundings are so different from those of a human that mutual understanding is not possible. The statement makes a valid point in an amusing and thought-provoking way. But, taken at face value, the statement is clearly incorrect. If an English-speaking lion came up to you and said that he was hungry, would you stand there wondering what he meant? I think not. I think most of us would get the message—and leave immediately. An English-speaking lion can communicate with humans, at least at some level. I once communicated with a coyote—and he couldn't even speak English. We eyed each other warily and slowly moved off in opposite directions. Mutual recognition is a kind of communication.

Years ago when one wrote a series of instructions in, say, the Fortran programming language, they had to be compiled. What the compiler did was translate the Fortran sentences into machine language. Once that was done, the program was run, and the machine carried out the instructions and gave back the results. So here we had a human (the programmer) and a machine (an electronic alien) communicating via a language that was foreign to them both. It would seem that the basic question concerning human-alien communication is not whether such communication is possible, but rather what level of communication is possible. Can we go beyond mutual recognition and, if so, how far?

What I find most curious is the opinion, expressed to me often and by many highly educated people, that this kind of communication is not only possible but also very easy. There are two ways people justify this opinion. First, there are those who believe that all we need do is exchange pictures and, somehow, that will enable us to say whatever we want. But if that were the case wouldn't it be trivial to communicate with other humans? So why do people make the effort to learn foreign languages? And does anyone seriously believe that when two world leaders meet to confer they leave their (human) translators at home and just exchange pictures?

A striking example of this kind of thinking occurred when I gave a lecture to a group of space scientists. I was describing how we might go about communicating the gram to an alien race (there is good reason, as we shall see, for finding some way to accomplish this). One man kept insisting that all we need do is show a picture of a balance scale. He kept interrupting the lecture and, finally exasperated, I turned to him with my hands out as though they were the pans of a balance scale and said, "Okay, here's a balance scale. Now tell me how that defines the gram?" He stared at me for a moment, announced that he had a meeting to attend, and fled from the room. Those who believe that pictures will solve the problem rarely carry their thinking very far. If they did, they'd soon see that the problem is not so easily solved. I think pictures will play an important role here, but they must be used in conjunction with language.

The other common justification given by those who think the problem is easy is that, they say, the code-breaking machines possessed by the NSA or the CIA will immediately translate any alien language. But if so why, after the tragedy of 9/11, did the FBI ask for more people (not machines) who were fluent in the languages spoken in the Middle East (*Arizona Daily Star* 2002)? And again, why do world leaders, when they meet, bring along human translators instead of just a laptop? The code-breaking machines are wonderful things and they do well what they were designed to do, but what they do is limited. They haven't helped much in translating the language of the dolphins.

I think the human race and an extraterrestrial one can learn to communicate. This is just my opinion. I certainly don't think such communication will be easy. The real question, as I have already said, is just what level of communication can we realistically hope to attain. The answer to that, of course, depends on the nature of the society we contact.

Our present means of searching for ETI involves the radio telescope. This method, of course, puts limits on the kind of society we might find. We will only detect societies that can send radio waves over interstellar distances.

So we are really searching for extraterrestrial technology, not extraterrestrial intelligence. There is no value judgment involved here. We would love to contact a race that emphasizes music, or art, or philosophy, or literature but, unless that race can send radio waves over long distances we will not detect them. Perhaps someday we'll have the means to detect such societies but, at present, we simply cannot do so. So any race we contact will have something like the radio telescope and hence a fairly sophisticated technology. Can we use this as the basis for some level of communication? This is an idea that we shall return to later on (Chapter 11).

Since our nearest neighbors will be light-years away, communicating with them will involve large intervals of time. Any message we get could, conceivably, come from a society that no longer exists. Such a message, if we can interpret it, can still be very valuable. The ancient Egyptians, the Greeks, Romans, Mayans, etc., all lived long ago and yet we still learn from them. They don't teach us how to cure all diseases, or how to eliminate highway congestion, or how to reverse global warming, but what they do teach us is valuable nonetheless. Learning about an alien society would be similarly enriching. We'd learn something about the nature of life, the nature of intelligence, and, perhaps, the nature of societies.

It is highly unlikely that we will learn how to cure all human diseases because the aliens won't know anything about human biology. They won't be able to tell us how to achieve world peace either, because they will know nothing about human history nor will they know anything about the complex structure of human societies. But still, we'd like to know about them and, perhaps, come to understand them a little bit. I think any intelligent race would want to know about us, our society, our world, and its biosphere.

Exchanging such information extends the lifespan of a society. It is a way to pass on a legacy to the intelligent life in the galaxy, a legacy that gives a deeper meaning to our existence. At some point every star dies and when the sun dies so will all human accomplishment. An exchange with an alien race will make it known that we lived and learned and did things; that we were aware of the beauty and majesty of the universe and recognized our role as beings that could experience and appreciate this beauty and majesty.

Perhaps this is the ultimate goal of SETI, to make our existence known and to pass on something of what we learned, something of what we were, to the other intelligent societies in the galaxy. The ancient societies of Earth have passed down such knowledge to us and this has greatly enhanced our understanding of what it means to be human. Our lives are richer because of this, and the ancient societies, to some extent, live on in us. Why not something like this on a galactic scale? Yes, I realize that if there is no one out there, then this idea is not worth much.

There is, however, one aspect of interstellar communication that goes beyond "ancient to modern" human communication. Since any society we contact will have a fairly sophisticated technology, it is not too much to suppose that they have an understanding of physical science. Using this as a basis, we might (for the benefit of some critics of SETI, let me be clear: I said "might," meaning "maybe" or "perhaps," not "certainly") be able to impart precise information about our sun, about our planet, and our solar system to an alien race, and they might be able to give us similar information about their star, their planet, their solar system. Such as exchange would greatly enhance our understanding of stellar physics, geophysics, celestial mechanics, perhaps even meteorology.

I have glossed over some tough questions here. The temperature of our sun is measured in degrees. How would an alien race know what our degree is? The energy the sun emits can be measured in calories; how would an alien race know what our calorie is? The mass of our planet can be measured in kilograms; how would an alien race know what a kilogram is? It was questions like these that led a colleague, linguist and logician Richard T. Oehrle, and I to try to find ways to communicate each of these units to a race that knows nothing about human conventions but does know something about science. The advantage in doing this is, of course, that communicating our units then makes it possible to exchange precise, not just qualitative but quantitative, scientific information. Now, before the critics start their predictable chanting of the obvious, let me admit that I can't know if this method, or any method, will work, but it might and since it might it is worth further discussion.

I should mention that Oehrle and I believe that a language based on science might be of value even if alien societies don't exist. Such a language might play a role in the development of the next generation of computer languages. By talking about aliens, however, we were able to focus our efforts and avoid making unwarranted assumptions. This technique (i.e., making up a problem and then trying to solve it) is often useful in doing research.

REMARK: Continuity of Space, Area, Integral Calculus and the Founding of Carthage, Line Integrals and the CAT Scan

Communication requires a shared perception of at least some phenomena. Our understanding of motion is based on our perception of space and time as continuous entities. Differential calculus combines these perceptions, giving us the mathematics of motion. We have already mentioned that the fundamental problem of differential calculus is finding the instantaneous velocity of an object. This can be formulated as a geometric rather than a physical problem. In connection with differential calculus, geometry and physics each give insight into the subject. The fundamental process here is called, as we have noted, differentiation.

The other major branch of calculus is concerned with a kind of inverse of differentiation called integration. Here our perception that space is continuous enables us to assign precise meaning to our intuitive understanding of area and volume. The integral calculus enables us to make these notions precise. The two operations, differentiation and integration, don't quite "undo" each other; they are not exactly inverses. For those interested, if one integrates a function and then differentiates the result, one gets the function back. But if you differentiate a function and then integrate the result you get the function plus an arbitrary constant. This may seem like an unfortunate nuisance but that is not so. In many applications these constants have important physical meaning.

The fundamental problem of integral calculus is finding the area under a curve. To see that there is a problem here, first recall that a square with side of length s is assigned area s^2. To find the area of some other region, say a circle, you fill it with squares (all of the same size) and add their areas. This gives an approximation to the area of the region. You then fill it with smaller squares and add their areas. This gives you a better approximation to the area of the circle because the smaller squares "fit" better. Now we repeat the process. Since space is continuous, the smaller we make the squares the closer we get to the area. The numbers obtained in this way "converge," meaning that they get closer and closer, to a fixed value that we take as the area of the circle. For a circle of radius r this process leads to the well-known formula πr^2. Ancient people found formulas for the areas of some common shapes by this processes of approximation. Integration is a formalization of this idea.

The fundamental theorem of calculus relates the two processes of integration and differentiation. It is a remarkable result with many far-reaching consequences. This is the kind of subtle fact we would look for in the mathematics of an alien race. Without this theorem the process of integration, which is a very reasonable way to treat area and volume, would lead to unwieldy, highly complicated, calculations. The fundamental theorem enables us to avoid these difficulties and still get the answers we want.

The importance of area was recognized even in ancient times. There is a historical incident that illustrates this point. Like many often-told historical incidents it might not even be true (did Washington really chop down a cherry tree?). But it is a good story anyway.

It seems that Dido, a Phoenician princess, was forced to relocate to northern Africa. Those already there, however, were reluctant to sell her any land. But she had a small army with her, and the northern Africans didn't dare be too rude. They tried another tactic: make the land so expensive that she'd give up and leave. Rather than leave, however, she purchased, for a considerable sum of money, all the land she could *enclose* in an ox-hide. Dido was no fool; she had a trick up her sleeve. She cut the ox-hide into thin strips, tired the strips together to make a long rope, and demanded all the land she could enclose with her rope.

Here is a problem in the subject now known as the calculus of variations. Of all the possible shapes she could enclose with the rope, which one contained the most area? To see that there really is a problem here, suppose that the rope is one hundred feet long. You could enclose a rectangle having two sides forty-nine feet long, and two sides one foot long. The enclosed area is then forty-nine square feet. But with the same hundred feet of rope you could enclose a square having each side twenty-five feet long. The square has the same perimeter as the rectangle, one hundred feet, but much more area; twenty-five times twenty-five which works out to 625 square feet.

It turns out that the maximum area, for a fixed perimeter (a rope of fixed length), is that enclosed by a circle. According to legend, Dido made a circle with her rope, claimed the land enclosed in it, and founded the city of Carthage upon it.

Area, and its three dimensional analog volume, are fundamental properties of space. They arise in ordinary life and in many areas of science. Here is another set of concepts we would expect to share with an alien race.

A formal definition of the integral didn't come until 1854 when Riemann gave one in a paper on trigonometric series (Chapter 7). In many cases it has a straightforward interpretation as the area under a curve, or the area between two curves. There is an important generalization of this process called line integration; it really is integration along a curve. This has no obvious geometric interpretation, but it does have a physical one.

Imagine moving an object along a curve. To do so a certain amount of work must be done. The line integral along the curve gives the numerical value of that work. This is important in the theory of fields (Chapter 4).

Sometimes the work done depends only on the point where the object starts and the point where it ends up. Mathematically this means that the line integrals along any two curves joining the points have the same value. Fields with this property are called "conservative"; an example is the gravitational field of the Earth.

Given a family of curves you sometimes know the value of the line integral of a function along each member of the family. You then try to find the function from this information. This is known as Radon's problem. The mathematics involved in solving it is at the heart of the CAT scan which, of course, is an important tool in modern medicine.

These applications make it seem that an alien race would share the idea of a line integral with us. This is by no means certain, especially since the concept has no immediate geometric interpretation. Integrating along a curve makes sense when you've seen the definition of the integral given by Riemann, but an alien race may have very different ideas about area and how one should define it.

Chapter 11

Languages

There have been, to my knowledge, only two[1] attempts at creating a language suitable for extraterrestrial communication. The first of these, called "Lincos," is due to the Dutch mathematician Hans Freudenthal (1960). Lincos is a contraction of the words "lingua cosmica" which means, of course, "cosmic language." This was published as a book with the subtitle: "Design of a Language for Cosmic Intercourse." English was not his first language and so Freudenthal was, perhaps, unaware of how this subtitle might be interpreted; especially by college students. When I mention the book to my classes the first question they ask is: "Does it have pictures?"

The second attempt is one I made in collaboration with my colleague Richard T. Oehrle (DeVito and Oehrle 1990). Humans and any alien society we contact will share the same physical universe. So, we wondered, might it not be that a language based on physical science can be understood by both parties? Maybe the physical universe itself provides the basis for a language suitable for extraterrestrial communication, a kind of cosmic Rosetta stone. With this thought in mind, we tried to construct such a language.

It should be obvious that there is no guarantee that either of these languages will be understood by an alien race. We may contact beings who are so different from us that little communication beyond mutual recognition is possible. These languages, Lincos and the other one (let's call this the D-O language), are simply attempts at showing how we *might* be able to communicate with an alien race. When Oehrle and I began work on this project we found that any suggestion could be "shot down." This was so easy that it rapidly became boring. It is also very easy to come up with arguments "proving" such communication is impossible. This too, rapidly became boring. We decided that it was much more constructive to work out an idea in detail and see if that led to someone coming up with a better idea, or a real improvement on what we did. That would be progress. SETI is, after all, an ongoing project. Our searches become more sophisticated as new, more sensitive methods of listening become available.

Our ideas on how we might communicate with an alien society should also evolve. These first attempts at "solving" the communication problem should be thought of as just that—first attempts. If we are serious about SETI, and I assume anyone involved in it must be, then we should plan ahead and try to have some idea of what we will do if/when the project succeeds; some examples of the kinds of planning I have in mind can be found in works by Harrison (1997) and Michaud (2007).

I am often asked why two mathematicians (myself and Freudenthal; Oehrle is a linguist and logician) were involved in language construction. What does such a project have to do with mathematics? This brings up an interesting point. Everyone knows that chemistry, for example, is an active area of scientific research. A person who obtains a bachelor's degree in chemistry has every right to be proud and can look forward to many exciting employment opportunities. If, however, that person wants to do research in chemistry, then he or she must go on to graduate school, learn a great deal more and, eventually, write a dissertation.

This must be based on research that he or she carries out under the direction of someone familiar with the problems that are of interest to modern chemists. Saying this in no way disparages the bachelor's degree or the intelligence of one who only has that degree. Obvious? Sure! But change the word "chemistry" to the word "mathematics" and, suddenly, everything I've said becomes unfamiliar, unbelievable, and even controversial.

There are still many who believe that calculus, the mathematics of the eighteenth century (some would say the seventeenth), is an active area of research. Those who have studied mathematics for a few years as undergraduates are convinced that they know all about the field (no one knows all of mathematics) and if you tell them there is more to learn, they become angry and defensive; they seem to feel that their intelligence is being questioned. Of course calculus, trigonometry, and arithmetic are all still useful subjects, but they are not areas of mathematical research. What many focus on is the skill aspect of mathematics. If you do lots of multiplication problems (or invert lots of Laplace transforms) you become very adept at this process.

You might find that you even multiply (or invert) faster than the average mathematician because he or she rarely multiplies numbers (and rarely inverts Laplace transforms). He or she is busy keeping up with the literature in the area that he or she is active in, and, given the pace of modern research, this is a full-time job.

What I am saying is that there is a profession called mathematics, and its practitioners spend their time creating *new* mathematics. The subject has a vast and rapidly growing literature, and becoming an active researcher requires graduate training and the profound commitment required of any profession; just like chemistry, physics, or astronomy. Surprisingly, very few people are aware of this, which causes lots of confusion and, sometimes, needless animosity. Most people associate mathematics with doing lengthy calculations; the kind of thing we now usually have computers do. The ability to carry out such calculations is an admirable skill, but it is not the same as expanding the realm of mathematics to include new areas.

Another common problem is that people often assume that whatever mathematics it is that they are using must be of interest to present-day mathematical researchers. If they are using tensor calculus or Bessel functions, then tensors and Bessel functions must be an active area of mathematical research. Unfortunately, this is rarely the case, as each field has its own set of current problems and areas of active interest; the various sciences are almost never in sync.

There is one other common assumption that causes endless confusion. This is the belief that mathematics is about the "real" world. Mathematics, like music, is a world unto itself. It is a world of abstractions and idealizations, a world apart from that of the every day. Even the natural numbers, 1, 2, 3, . . . , are abstractions. You've never kicked over a rock and seen a five scurry away, and no prospector, exploring a remote valley, ever came across a rich vein of fours. Bizarre things can happen in the world of mathematics. A particularly striking example of this is the so-called Banach-Tarski-Hausdorff "paradox." This says that given a sphere the size of a pea one can slice it into a finite number of pieces, reassemble the pieces, and get a sphere the size of the sun (Wapner 2005)!

Results like this one show why, in the world of mathematics, definitions must be carefully formulated and standards of rigor must be very high, much higher than those necessary in the "real" world. We have seen how scientists who naïvely assumed that what they saw on Mars was analogous to what they saw on Earth were led to erroneous conclusions. They learned to be more careful. Similarly, mathematicians in earlier times who reasoned intuitively learned, sometimes by bitter experience, that they had to be more careful than that. Mathematicians insist on rigor not just to annoy their colleagues (that's just an added bonus); they do it because they have learned that they have to. The world of mathematics, unlike the world of everyday life, requires it.

In the nineteenth century, while working on a problem in mathematical analysis, Georg Cantor was led to investigate the properties of infinite sets (see the Remarks in Chapters 1 and 2, and also Appendix I). His results were astounding and, among other things, he introduced the transfinite numbers. The first transfinite number is usually denoted by the Hebrew letter *aleph* on which a subscript of zero is attached $\aleph_0$. This number is bigger than any of the numbers 1, 2, 3, . . . but Cantor found that there are numbers beyond even this one. This work is beautiful, intellectually stimulating, but highly counterintuitive, and when it was first published it created a nasty controversy (Dantzig 2007: 215–238). Moreover, as people delved into these ideas, a number of disturbing paradoxes arose (these are discussed in the next chapter). All of this drew the attention of the mathematical community to the foundations of their subject. Research in those foundations goes on to this day.

What does all this have to do with languages and, more specifically, with SETI? To begin to answer that question let me quote from Freudenthal's introduction to *Lincos* (1960: 11): "So far I have not yet mentioned the logistic language created by G. Peano and perfected by B. Russell and A. N. Whitehead. There have been earlier attempts, but Peano was the first to design a linguistic pattern more adequate to mathematical reasoning than common language." The Italian mathematician Guiseppe Peano wanted to devise a logistic (i.e., logic-based) language that could be used for mathematical exposition, thus avoiding the necessity of learning to read English, French, German, Russian, etc.

Whitehead and Russell attended the world congress of mathematicians held at the University of Paris in 1900.

There they met Peano, and the three men spent considerable time together discussing logistic languages. It was recognized that some of the paradoxes I mentioned before were the result of an imprecision in the natural languages (the term "natural language" simply means any language spoken by human beings).

The two Englishmen realized that use of logistic language enabled one to avoid these paradoxes. Is this so important? We use natural languages all the time with no ill effects, so why bother to construct a logic-based language? It wasn't personal preference but rather experience that forced mathematicians to the realization that, in dealing with foundations, logistic languages played a crucial role.

Also at this conference a young German, one of the greatest mathematicians of all time, gave a list of problems to challenge the mathematicians of the coming century (no, it wasn't Einstein). The very first of these had to do with Cantor's work. Thus there was renewed interest in the foundations of mathematics and a surge of research in the area. The young German was David Hilbert. It is interesting that some say his list contained twenty-three problems while others say it contained twenty-eight. This is not surprising. Everyone knows that there are three kinds of mathematicians—those who can count and those who can't.

Eight years after the Paris conference Russell and Whitehead published a monumental work called *Principia Mathematica.* In it they devised a logistic language and used it to systemically develop the foundation of mathematics. Soon others followed their lead, and logistic languages became a basic tool in foundational research.

I might mention that this esoteric research became very useful some decades later when people began developing computer languages. Mathematics has this annoying habit of becoming useful sometimes long after it was first created. The theory of functions of a complex variable was extensively developed in the nineteenth century (see the Remark below). This subject is now of fundamental importance in some areas of electrical engineering. There was no electrical engineering in the nineteenth century (of course, figuring out how to apply this area of mathematics to electrical problems involved some brilliant engineering). The tensor calculus of Ricci and Levi-Civita, developed late in that same century to deal with problems of geometry, was used by Einstein to formulate his brilliant general theory of relativity in 1915.

Numerous other examples of this can be given but to do so would take us too far off our current subject.

But from the point of view of those interested in SETI this shift in emphasis from Peano's idea of a logic-based language that would supersede the vernacular in mathematical exposition to using such languages in foundational exploration, was unfortunate. The fact that such languages could be used for more general communication was forgotten. It was Freudenthal who tried to return to Peano's original idea by suggesting that we use a logistic language for extraterrestrial communication.

Does this mean that Freudenthal believed aliens exist or that UFOs were alien spaceships? Let me tell you what he said: "Of course I do not know whether there is any humanlike being on other celestial bodies, and even if there were millions of planets in the universe inhabited by humanlike beings, it is possible that our nearest neighbor lives at a distance of a million light-years and, as a consequence, beyond our reach" (Freudenthal 1960: 14).

These are not the words of a "believer." They are the words of someone who has an understanding of the physical reality we are confronted with. So what was he trying to do? Again let me give you his words: "My purpose is to design a language that can be understood by a person not acquainted with any of our natural languages or even their syntactical structures" (Freudenthal 1960: 13).

His use of the term "person" might be misleading. He explains his meaning later: "On the other hand I shall suppose that the person who is to receive my messages is human or at least humanlike as to his mental state and experiences. I should not know how to communicate with an individual who does not fulfill these requirements. Yet I shall not suppose that the receivers of my messages must be human or humanlike in the sense of anatomy and physiology" (Freudenthal 1960: 14).

Freudenthal was interested in using logic-based language for general communication. Why then, did he bring in aliens? He explained:

> One can imagine numerous ways of tackling the subject. After several unsuccessful attempts I finally became convinced that it is just the difficulty of choice which causes the trouble, and that the only thing which matters is to find a starting point. Seeking in history how analogous situations were met, I came to the conclusion that one should start with a concrete, sharply-defined and rather narrow problem. (Freudenthal 1960: 12)

I might mention here that one of the ways that mathematicians create new mathematics is by investigating "good" problems. Now the guy who wants me to do his income tax might think that he is giving me a good problem. He isn't, because in solving this particular problem all you learn is the answer. Most problems are like that, and, consequently, most problems are of interest only to those who want to know the answer. A really good problem leads to new ideas and sometimes whole new fields of mathematics.

Such problems are rare and, sometimes, they may seem weird, pointless, or at least very strange. For example the famous "last problem of Fermat" was concerned with whether or not a certain class of equations ($x^n + y^n = z^n$ for $n \geq 3$) had whole number solutions. Why would anyone care? That problem, however, led to whole new fields of mathematics, and so, as strange and pointless as it might seem, it was a very good problem. It was around for over 350 years before Andrew Wiles solved it in 1994. Incidentally, the equation above, when n=2, has lots of whole number solutions. We could take x to be 3, y to be 4, and z to be 5; or take x to be 5, y to be 12, and z to be 13, and there are many others. Fermat was reading a book about this when the problem occurred to him. He even thought he had a proof that no whole number solutions existed when n was three or greater. He made a note in the book saying he had a wonderful proof but it would not fit in the margin. When he died the book with its marginal note was found and attempts to prove the result began. Andrew Wiles proved him right many years later.

"In order to narrow the problem, I propose to exclude or at least to restrict excessively opportunities for showing," said Freudenthal (1960: 14). "I shall use showing as little as possible as a means of explaining the meaning of linguistic expressions." Later on he states, "As a linguistic vehicle I propose to use radio signals of various duration and wave length. These two dimensions will suffice" (1960: 14). Notice how these last two statements make Lincos a possible language for communicating with the kind of societies that are currently within our technical capabilities to reach.

In both Lincos and the D-O language, we start with the natural or "counting" numbers. One beep for 1, two beeps for 2, etc. We are focusing the attention of our correspondents on simple concepts that, since they have something like the radio telescope, are probably familiar to them. Using the properties of these numbers we try to communicate the basic components of a simple language; e.g., things like "and," "or," negation, and implication. Freudenthal then goes on to discuss human behavior.

Oehrle and I took a rather different path. Since the aliens probably know something of physics and chemistry—yes, this is an assumption and not an incontestable fact—we felt we could use this knowledge to more rapidly reach the point where interesting information could be imparted. As I have already mentioned the distances, and hence the time, involved in this kind of communication, means that our messages will be pretty much disjoint. This won't be a dialogue in the usual sense. So what we must do is teach the aliens a simple language and then use that language to tell them more and more about us, but we must do this in such a way that each subsequent part of the message enhances and enriches the language. Now before I am accused of some kind of arrogant human chauvinism let me add that, I think, the aliens will do something similar, i.e., teach us their idea of a cosmic language and use it to tell us more and more about them. Perhaps I am being naïve, but this kind of an exchange seems, to me, to be the only realistic one possible given the physical limitations we are faced with. The constraints of special relativity (see Chapter 14) severely limit our options.

Many critics seem to think that those of us who try to solve the language problem are unaware of the many difficulties involved. Others seem to believe that Oehrle and I think our work solves the problem completely. Nothing could be further from the truth. The approach we took is based on numbers because we couldn't think of anything simpler. Our work is the best that we could come up with, but better ways may certainly exist. What we did learn from our attempt is that the construction of a language based on science is more subtle and more delicate than we at first supposed. Ideas for simplifying some aspect of the language invariably led to complication at some other stage and attempts at avoiding chemistry and using some other science as our starting point, physics say, were unsuccessful. This does not mean that it cannot be done. It just means that we were unable to do it. We tried to base a language on science because we think that any society we contact will know some science since we can only detect societies that have a sophisticated technology. I would certainly welcome constructive suggestions from others who see ways to improve our work or who see totally different ways to approach the problem.

REMARK: Real Numbers as the Basis for Calculus, Complex Numbers and the Calculus of Complex Functions, Complex Integration, and Whether Mathematical Objects Are Real

Russell and Whitehead wanted to develop all of mathematics from a few axioms. This is a difficult undertaking that involves finding ways to deal with many highly technical matters and many philosophical questions. But SETI researchers usually assume that any race we contact, any race with the ability to engage in inter-stellar communication, will be familiar with the natural numbers. So the question arises as to how much of human mathematics we can communicate to such a race.

Karl Weierstrass, one of the great nineteenth-century German mathematicians, was the first to recognize that the foundations of calculus had to be based on an understanding of the properties of real numbers. But the real numbers are rather mysterious. They contain all the rational numbers and, as we have seen, they also contain certain numbers that are not rational. Where are these irrational numbers and how do they fit in among the rational numbers? In earlier times this question was usually answered by stating that the real numbers were in one-to-one correspondence with the points on a line (see below).Thus the question was answered by appealing to our geometric intuition.

This is unsatisfactory for philosophical reasons, and a better answer was given in the nineteenth century by two Germans. What they did was show how one could construct the real numbers from the rational numbers without bringing in extraneous geometric ideas. The two were Georg Cantor and Richard Dedekind. They solved the problem independently and by quite different methods.

Starting with the natural numbers, one can communicate the whole numbers and the rational numbers (fractions) very easily. Using the method of Dedekind or that of Cantor, one can then communicate the real numbers. Once we have done that we can, following Weierstrass, communicate all of mathematical analysis. *So, in principle, virtually all of our mathematics can be communicated to any race that understands the natural numbers.*

We can picture the real numbers as a line. Imagine a horizontal line; pick a point on that line and call it zero. Choose an arbitrary unit and mark a one at the point one unit to the right of zero, and a minus one, one unit to the left of zero.

Continue this process. Each point on the line corresponds to a unique real number, the distance of that point to zero if it is to the right and the negative of that distance if it is to the left, and, since a line is "continuous," each real number should correspond to some point on the line; there can't be any "holes." This construction is called the real line, or the number line, or the mathematical continuum. It is the standard model for time in physics.

This picture is simple and useful but it hides the rather sophisticated structure of the real number system. They are lined up but not at all like the beads on a string. Between any two distinct real numbers there is always a rational number, in fact, infinitely many of them. Between these two numbers there are also infinitely irrational numbers. So given a real number it makes no sense to speak of the "next" number, and removing a real number from the line leaves a gap but the gap has no edges.

These facts may give the impression that the rational numbers and the irrational numbers are equi-numerous;

that they can be put in one-to-one correspondence. This is not so.

The irrational numbers are far more numerous than the rational numbers (see Appendix I). Here are two infinite sets, one of which is far more "infinite" than the other.

It was during the renaissance that a new, richer number system was introduced. This is the system of complex numbers. A complex number can be written as a + bi, where a and b are real numbers, and i is the square root of minus one; so $i^2 = -1$. There is no real number with this property. The square of zero is zero, and the square of any other real number is positive. Complex numbers were used by Cardano in 1545 to solve a problem of algebra, and their arithmetic was worked out by Bombelli around 1572. No one took them very seriously even when Thomas Harriot, around 1630, showed that they were of fundamental importance in solving equations.

It wasn't until 1797, when a Norwegian surveyor named Wessel presented a geometric interpretation of complex numbers to the Danish academy of sciences, that people began to feel more comfortable with them.

We humans seem to feel better about abstract ideas when we can visualize them in some way. Wessel's idea was independently discovered in 1799 by the astronomer Bessel, and in 1807 by a bookkeeper named Argand. Ironically it is now often called the Argand diagram.

The complex numbers are of fundamental importance in modern electrical engineering and in certain areas of modern physics. As we have seen, they were investigated long before their applications were even dreamed of. The calculus of functions from the complex numbers to the complex numbers was developed extensively in the nineteenth century. The differentiation of these functions is a straightforward generalization of differentiation of real functions. Complex integration however is essentially line integration (Chapter 10). It is vitally important here, but as we said before, it has no obvious geometric interpretation. The calculus of complex functions has so many applications in electrical engineering and in physics that it is hard to imagine a society that has an advanced technology, and has the radio telescope, and yet knows nothing of this subject.

The geometric picture devised by Wessel enables us to apply trigonometry to the arithmetic of complex numbers. This enables us to compute roots of complex numbers. The roots of unity (the number one) are especially important in digital signal processing, something engineers interested in SETI know a great deal about.

Can we assume that an alien race will have found the complex numbers? The problem we keep running into with our questions, here and in the remarks made earlier, concerns the "reality" of mathematical objects. If they, in some sense, exist outside of our minds, then there is a good chance any intelligent race will have discovered them, because they are useful in solving real-world problems and in modeling aspects of reality. On the other hand if they exist only in our minds, then there is no reason to expect an alien race to know about them. The aliens might have their own ways of solving problems, even the practical problems that arise in engineering, that are totally disjointed from human mathematics.

Note

1. At a recent conference Professor Ollongren, of the University of Leiden in The Netherlands, told me of his book *Astrolingustics* which deals with the same problems treated in Lincos. At the time of this writing, Ollongren's book, to be published by Springer, was not yet in print.

Chapter 12

Paradoxes

This brief chapter is a digression, a break from the main theme of this book. It contains a short discussion of the amusing, frustrating, and, perhaps, annoying paradoxes of set theory alluded to earlier (see Chapter 11; a more technical discussion can be found in Enderton 1977: 5–6).

These paradoxes are of two types: logical and semantic. Among the first kind the most well-known is, perhaps, the one due to Bertrand Russell. It might be best to start with a story that illustrates its difficulty.

There is an imaginary town where all men are required to shave, or be shaved, daily. The town is so small that it has only one barber and he shaves those men, and only those men, who do not shave themselves. Now, who shaves the barber?

If you say he shaves himself then you must conclude that he is not shaved by the barber; because the barber shaves only those men who do not shave themselves. But he is the barber! So if he shaves himself then you must conclude that he does not shave himself—a clear contradiction.

Okay, so maybe he does not shave himself. If that is the case then he must be shaved by the barber. Again, since he is the barber, we have another contradiction.

To bring this a little closer to mathematics recall the idea of a set (Chapter 1). The set of all sets is a set (see below) and, as such, it belongs to itself. But the set of all books is certainly not a book, so this set does not belong to itself. If we let S be the set of all sets that do not belong to themselves, like the set of all books or the set of all people, then when we ask whether or not S belongs to S, we get into the same difficulties we had with the barber. If S belongs to itself, then it can't belong to itself and if S does not belong to itself then it does. If you are going to develop the foundations of mathematics from set theory, then such paradoxes must be avoided. The hard question is just how does one do that?

An interesting example of a semantic paradox is one due to Berry. To present this one we must first recall the natural numbers, 1, 2, 3, 4, ... A surprisingly far reaching property of these numbers is this: *Any non-empty set of natural numbers has a smallest member.*

The smallest even number is, of course, 2. The smallest seven digit number is 1,000,000, and other examples can be easily given. Suppose we let T be the set of natural numbers that can be described in fewer than twenty-five words from a standard English dictionary. There are only so many words in the dictionary, and only so many ways of combining them, so T is a finite set. Since there are infinitely many natural numbers, the set of natural numbers that are not in T is non-empty (it is actually infinite) and so it has a smallest member. *This is the smallest number that can not be described in fewer than twenty-five words from our dictionary.*

But this last sentence describes this number in only nineteen words! The use of logistic languages enables one to avoid paradoxes like this one.

A careful treatment of set theory requires that we limit the notion of set; Russell's paradox, above, shows this. There is an even simpler paradox that arises when we are too casual in our use of the term "set." Let S be a given set. We can collect together all subsets of S (Chapter 1) and, in this way, we get a new set; often denoted by P(S). If S has n members it turns out that P(S) has 2^n members; this explains our notation, P(S) is called the power set of S. So a set with three members has $2^3 = 8$ subsets, and a set that has 10 members has $2^{10} = 1{,}024$ subsets. What happens when S is an infinite set? One can show (see Appendix I) that there is no one-to-one correspondence between S and P(S); P(S) is always bigger than S. But suppose S is the set of all sets! Doesn't it follow that then P(S) coincides with S? So it seems that the set of all sets is not a set!

REMARK: Group Theory in Algebra and Geometry

Modern algebra is concerned with abstract structures extracted from a variety of concrete examples. This involves a higher level of sophistication than found in the more elementary parts of mathematics.

The theory of groups, in particular, first arose in connection with equations and arises again in the study of the ways a geometric object can be transformed without altering its shape. We give a brief discussion of the development of this theory here.

The investigation of equations is a fascinating, but rather complicated, chapter in the history of algebra (Stewart 2007: ch. 13). As with all of mathematics, developing good notation goes a long way in helping clarify many of the problems in this area and in illuminating the methods used to solve them. Historically, many of the problems arose before the notation had been devised, making these problems much more difficult for the early workers than they are for us today. Sometimes the contributions of early generations of mathematicians are dismissed as easy by people unaware of the fact that modern notation is what makes them easy—notation that wasn't available to previous generations.

A technique for solving equations of the second degree—quadratic equations—was known as early as 1,700 BCE.

This involved the four arithmetic operations and the extraction of a square root. It wasn't until the Renaissance, however, that the general cubic equation (an equation of degree three) was solved; certain special cubics had been solved geometrically much earlier. It was in connection with cubic equations that the complex numbers were discovered. The solution of the cubic involves the four arithmetic operations and the extraction of a cube root. Equations of the fourth degree were also solved during this period. Here the technique involved solving a cubic, the resolvent cubic of the given equation, and two quadratics. So again one uses the four arithmetic operations and root extraction. Equations whose solutions can be found by using the four arithmetic operations and root extraction are said to be "solvable by radicals."

For many years people tried to solve the general equation of the fifth degree (the quintic) using these techniques; it can be done in some cases. It was eventually realized that this might not be possible for the general quintic, and a proof of this was given by the Norwegian mathematician Abel in 1823.

Let's be clear. The question was whether or not the general equation of degree five could be solved using only the four arithmetic operations and root extraction. Abel showed that this was not possible. Of course these equations can be solved, but to do so one must use other tools.

The question arises: why did mathematicians insist on using only the arithmetic operations and root extraction in their attempts to solve equations? The reason seems to be purely historical. These methods worked for equations of degree two, three, and four, so why not degree five and beyond? It is important to note, however, that a great deal of mathematics that is now of great value in science and elsewhere was discovered in the course of trying to figure out which equations could be solved by radicals and which could not. One of the most important outcomes was the formulation of the concept of a "group."

Group theory is of fundamental importance in modern physics and in certain areas of chemistry (Mackey 1973). Take a crystal, for example: the different ways one can transform it, without changing its shape, leads to a group called the group of symmetries of the crystal.

A simple example is a rectangle. One can reflect it in a horizontal axis so that the top corners become the bottom corners and vice versa. One can also reflect it in a vertical axis so that its left-hand corners become its right-hand corners and vice versa. One can also rotate it through 180 degrees so that its top becomes its bottom and its left side becomes its right side, and, of course, you can leave it alone. These four "symmetries of the rectangle" constitute the group of symmetries of the rectangle. The group operation is simple. To combine two symmetries we apply them in succession. If we reflect in a horizontal axis and then reflect in a vertical axis the result is the same as rotating the rectangle through 180 degrees. At this point it may be a little hard to see the value of this construction. What have we gained by looking at the group of symmetries of the rectangle?

A good response was given by Harvard University mathematician George Mackey:

> The answer lies in the fact that many mathematical systems, including those which model the physical world, also have symmetries and symmetry groups and that the study of the structural and other properties of these symmetry groups provides profound insights into the more immediately interesting properties of these systems and the key to the solution of many important problems.

What does all this have to do with equations? First recall that there are six ways you can arrange three books on a shelf. Each arrangement is called a permutation, and it turns out that the six permutations form a group. An equation of degree three has three solutions, called its roots. The six ways these can be arranged give us the Galois group of the equation (Galois was a French mathematician who made remarkable progress in the study of equations before his death at the age of 21). By studying this group, which can be computed without knowing the roots, we can tell whether or not the equation can be solved by radicals. This can always be done for the case of equations of degree two, three, or four.

The group of an equation of degree five contains 120 members. The structure of this group tells you whether or not the equation can be solved by radicals.

Understanding the behavior of equations requires a high level of abstraction and quite sophisticated reasoning. This investigation led us to the far-reaching idea of a group which, since its introduction, has found many real-world applications. Will we share this with an alien race? The symmetries of a crystal are certainly real. But realizing that these symmetries comprise an algebraic object with remarkable properties of its own is quite a leap in abstraction; a leap the human race had trouble making. But symmetry in one form or another is prevalent in nature. There should, in principle, be many roads to group theory and the way an alien race deals with this aspect of reality may tell us a lot about them.

Chapter 13

The Universal Science

The world around us, and in fact the entire universe, seems to consist of two things: Matter and energy. Matter is usually defined as anything that occupies space and has weight. Energy is a little harder to define, but examples surround us: heat, light, electricity, sound, etc. For a long time it was believed that the bulk of the matter in the universe was visible because it either emitted or reflected light. Today astronomers have come to realize that a large portion, some say as much as 80 percent, of the matter in the universe is dark (Kaufmann 1994: 465, 490; see also Bennett 2001: 153–67). The nature of this “dark matter” is one of the great mysteries of modern astronomy; we really don’t know much about it yet. But let us concentrate on visible matter, since we do know something about this.

Visible matter exists in three forms called states: solid, like ice; liquid, like water; and gas, like steam.

As this familiar example illustrates, the same material can exist in all three forms. Water is pretty stable; heating it changes its state but not its chemical composition. Some things, when heated, will decompose, catch fire, or even explode. In other words, they change into something else. But even water can be broken down. Using a process known as electrolysis it can be decomposed into two gases. One of these is hydrogen, light and flammable, and the other is oxygen, the gas we breathe to stay alive. These cannot be broken down further. So we might classify matter into two types—those that can be broken down into something else, and those that can't. The latter are called elements.

Chemists have found that there are only 92 naturally occurring elements and that all earthly matter is made by combining these. Many are quite familiar, such as gold, silver, iron, copper, carbon, iodine, chlorine, etc. Some are rarely seen outside a chemistry laboratory but are found in drugs and food, such as sodium in table salt, potassium, calcium, strontium, and lithium (all found in drugs), and boron (found in boric acid).Elements are most often found in chemical combination with other elements. Such combinations are called compounds, as opposed to mixtures.

In a mixture of powdered sulfur and iron filings, the sulfur will burn and a magnet will attract the iron. But when these two elements combine chemically, the product no longer burns and is not attracted by a magnet. The two elements are not just mixed together but actually combine chemically to produce something new: A compound called iron sulfide.

When a sodium compound is sprinkled onto a flame, the flame takes on a bright yellow color. Potassium compounds turn a flame violet, while compounds containing lithium or strontium turn the flame red. Fourth of July fireworks displays are based on these facts. We have all seen how a prism will break white light into a rainbow of color. But the light from a flame that has been sprinkled with a strontium compound, when passed through a prism, gives a cluster of red lines and blue lines against a black background. If you use a lithium compound you get a red line, a yellow line, and two blue lines against a black background.

Each element, in fact, has a characteristic "line spectrum," obtained by passing the light you get by heating the element, through a prism. It is this fact that enables astronomers to identify the chemical elements present in a far away star (Ebbing 1987: 177). We have seen (Chapter 6) that the visible portion of the electro-magnetic spectrum starts at red (low frequency) and climbs to violet (higher frequency). The spectra of the elements in the light from many stars are shifted toward the red. This is due to the fact that these stars are moving away from us. By measuring this "redshift" astronomers can calculate the speed at which the star is moving.

Amazingly, the same 92 elements found on Earth, and only these, are found throughout the visible universe! Once scientists investigated the question of where the elements come from, this fact became somewhat less surprising (see below). Chemistry, then, is not just an earthly science. It is universal, and any aliens out there, if they study chemistry, are studying the same elements that our chemists here study.

Furthermore, the properties of these elements vary in a regular, periodic, way. It was the Russian chemist Dmitri Ivanovich Mendeleev who developed a tabular arrangement of the elements that clearly expresses the regular variation of their properties, listing elements with similar properties in vertical columns (Ebbing 1987: 48). My colleagues in chemistry tell me that it would be hard to overestimate the importance of this table, called the periodic table, for the development of their science. This table, or something like it, might be known to an alien race and could be the basis for some form of communication.

At some point scientists asked the question: Where did the elements come from? Each element has an atomic number: 1 for hydrogen, 2 for helium, etc. This is the number of protons in the nucleus of an atom of that element (Chapter 7). The first stars were formed from vast clouds of hydrogen gas. As the gas cloud grew the temperature at the core increased until the hydrogen ignited into a nuclear furnace. This is not ordinary burning. It is a thermonuclear process that produces incredible amounts of energy and causes the star to shine. The end product of this process is helium. If the star is massive enough, the pressure from its outer layers will cause the helium to ignite and this produces carbon, atomic number 6. As the star goes through its life cycle other elements are created—e.g., oxygen, atomic number 8; neon, number 10; silicon, number 14; and finally iron, number 26. When the star dies this material is thrown back into space, sometimes in a violent explosion which in itself causes elements of higher atomic numbers to form. Eventually, this material is recombined into another star, a star that might have planets around it. That is what happened in the case of our Sun. So the Earth and all life on it, including us, are all star stuff. We consist of material that was formed eons ago in the interior of some star.

There is another aspect of chemistry that is important in connection with SETI. Whether one has hands, claws, tentacles, or suction pads, one cannot manipulate individual atoms. So how do you make meaningful chemical calculations? Suppose you want to manufacture 100 pounds of iron sulfide. How much sulfur and how much iron should you use? To answer such questions chemists had to learn something about how atoms combine (i.e., is it one to one, or many to one?), they had to devise a system of atomic weights (oxygen is 16 times heavier than hydrogen, while carbon is 12 times heavier) and they had to know how many atoms were in a fixed weight of a given element or compound (e.g., 12 grams of carbon contains an Avogadro number of atoms; see the Remark below).

This crucial number, the Avogadro number, has been determined experimentally in many ways (Peaslee 1955), and this number, or something like it, must be known to any society that has a science of chemistry! Using this we can (maybe, perhaps) communicate our gram.

Chemistry comes into play in connection with other human units as well. Many substances melt and boil in a very distinctive way. To be specific, let's consider water, although this does involve pressure, because it is so familiar. Adding one calorie of heat to one gram of ice raises its temperature by 1 degree Celsius. This is true until you reach 0 degrees. Then any additional heat goes into changing the state from ice to water. It takes about 80 calories of heat to change one gram of ice at 0 degrees to one gram of water at 0 degrees. After that, adding one calorie raises the temperature 1 degree as before. But when we reach 100 degrees Celsius then, again, any heat added goes into changing the state. It takes about 540 calories to change one gram of liquid water at 100 degrees to one gram of steam at 100 degrees. This is why a steam burn is so much worse than a hot water burn.

This distinctive melting and boiling behavior is true of many substances and it is *not* a human invention or human convention; it is a fact of nature. Using this we may be able to communicate, to any society that knows some chemistry, our calorie and our degree. In our paper we used the melting points of silver and of lead since these, unlike water, are not affected by pressure (see Appendix III). To match up our temperature scales we must also find a temperature we have in common. But this again comes from chemistry. There is a real lowest point on any temperature scale, and it is called absolute zero.

As one can see, chemistry can help two races learn a great deal from each other. More specifically, they can learn each other's basic units of measurement. This would be a great step forward since it would then enable the societies to exchange *precise* scientific information.

The language Oehrle and I constructed is based on the facts of science outlined above. We think these facts—and they are facts—will be known to any society that has the radio telescope or, if one of them is unfamiliar, they can easily test what we say experimentally. In this way we think communication can begin and, fairly rapidly, progress to the point where interesting information can be exchanged. Of course this is an assumption. There are no guarantees here, but it is conceivable that there is a society out there that does know these things and will understand what we are trying to say. If the society we contact does not know these things or cannot figure out what our messages mean, then this method will fail.

As this kind of communication progresses we must at some point make use of pictures. Of course we can't know if the aliens we are in contact with can see and, even if they can, we can't know how they will interpret any picture they receive. Let us assume that they can see.

There is some justification for this:

> The eye has been "invented" three separate times on Earth—the cephalad eye, the insect eye, and the vertebrate eye. They all have different, totally independent evolutionary histories; yet each of the three organs has evolved to serve essentially the same purpose and all three have basically the same neural networks. So, it would seem that in any world where the optical spectrum band is important, there would be a good chance its inhabitants would develop a light-sensing organ with similar nerve structure. And, the same logic insists, in any world where brains could aid survival, creatures would, nature willing, develop them. (Blum 1990: 122; see also Darling 2001: 124–25)

If we follow the path suggested above we can use the language to caption the first pictures we send so as to teach our correspondents how these pictures are to be understood—we can show them which end is up. Presumably they will do the same for us so we can correctly interpret the pictures they send. At this point some serious psychological or cultural problems might arise. The pictures might show something we find disturbing, frightening, or even disgusting.

It might be wise to have any picture received carefully inspected by a team of anthropologists skilled in crosscultural work. They can then prepare the public for what they will see; in some cultures left-handed aliens might be troubling and this is just one potential problem.

Anthropologists will also play a key role in designing pictures that accurately present the human race to an alien one. They are familiar with the various "faces" of mankind and can best decide how to communicate these "faces" to our correspondents.

There is one advantage of a scientific approach to language that I have not yet mentioned. If the early messages contain mathematical and scientific information, they can be made public without causing undo fear. Many people will interpret a message from the stars in religious or moral terms. Some will see them as messages from God while others may see them as the work of the devil. In the twenty-first century we have already seen how religious fanaticism can turn deadly.

But if the early messages merely deal with the boiling point of ammonia, or the melting behavior of lead, then few will associate them with angels or devils. The human race will have the time, and the emotional distance, to come to terms with this new era in human history and accept the fact that aliens are not some spiritual manifestation.

The presence of "dark matter" in the universe was first inferred from the behavior of clusters of galaxies. Recall that visible light occupies a portion of the electro-magnetic spectrum from red on up to violet (Chapter 6). As we have already noted, if a star is moving away from us the spectra of the elements in that star are all shifted toward the red. It turns out that the light from other galaxies is also shifted toward the red. They are all moving away from us. This is how astronomers know that the universe is expanding.

In the 1930s the Swiss-American astronomer Fritz Zwicky measured the differences between the redshifts of the galaxies in the Coma cluster and the redshift of the entire cluster. This allowed him to calculate the mass of the cluster. Then by measuring the total amount of light coming from all the galaxies in the cluster he estimated the number of stars the cluster must contain to produce that light. He found the number of stars was far too few to account for the mass he had measured and concluded that much of the mass of the cluster must be dark.

Zwicky's result wasn't taken too seriously until the 1960s when Vera Rubin measured the velocities of stars in the Andromeda galaxy.

From these measurements she could, using Newton's law of gravity, estimate the mass of the galaxy that was within the star's orbit. One would expect that, for stars located far from the center, the mass estimate would be about the same, because pictures of the galaxy show that the majority of the stars are in the central region and only relatively few stars lie on the outskirts. To her surprise the measurements showed that there was lots of mass out beyond the center. Numerous studies have now shown that galaxies consist mostly of dark matter and that matter is located far from their centers.

The nature of dark matter remains pretty much a mystery. Some of it might consist of "failed stars" that are too dim to be seen. These are sometimes referred to as MACHOs, for "massive compact halo objects." There is other evidence for the existence of dark matter and some of it implies that at least some dark matter cannot be made of ordinary protons or neutrons. This form of dark matter is referred to as weakly interacting particles, WIMPs.

It has long been known that, as we noted above, the universe is expanding. The natural question implied by this fact is this: Will the universe expand forever, or will it eventually stop and, perhaps, contract, ending up in a "big crunch"? There is a type of super nova (exploding star) called a Type 1a. It has been found that all novae of this type have pretty much the same intrinsic brightness, and so their apparent brightness depends only on their distance from us. Observations of type 1a super novae in distant galaxies, however, have shown that, to the surprise of many, they are dimmer than they would be if the universe were expanding at a constant rate. Thus the expansion of the universe is actually accelerating. The cause of this acceleration is attributed to the presence of "dark energy." This mysterious repulsive force is the subject of much current investigation.

REMARK: Atomic Weights and the Avogadro Number

In connection with extraterrestrial communication, we must find ways to communicate many things we take for granted. What is hydrogen, helium, etc.? There is a function that assigns to each atom the number of protons in its nucleus; this is its atomic number. If this is communicated, then hydrogen is the set of all atoms with atomic number one, helium the set of all atoms with atomic number two, and so on. Once these have been defined we can arrange them in a table, the famous periodic table found by Mendeleev mentioned above. It displays the elements so that those with similar properties are in vertical columns, and the properties of those in a horizontal row vary in a way that conforms to observations. Perhaps this table will be known to any race that has a science of chemistry. If so, then the meaning of our sets will become clear.

Avogadro's law is of fundamental importance in chemistry. Before we can present it we must first say something about the system of atomic masses, often called atomic weights. It was observed, for example, that two grams of hydrogen combined with sixteen grams of oxygen to form water. The formula for water is H_2O, hence the masses of individual hydrogen and oxygen atoms are in the ratio of

1:16. Experiments of this kind, using many compounds, led chemists to an understanding of the relative masses of all atoms.

A system of atomic masses was developed by assigning a mass of twelve to a certain isotope of carbon and computing the masses of the other atoms from this. All atoms of carbon contain six protons, but they can differ in the number of neutrons they contain (Chapter 7). These are called isotopes of carbon. The isotope containing six neutrons is the most common and is taken as the standard. Avogadro's law may be stated as follows: *One gram atomic weight of any element contains the same number of atoms.*

So, twelve grams of carbon contain the same number of atoms as sixteen grams of oxygen or one gram of atomic hydrogen. This number, the Avogadro number, is 6.023×10^{23}. There are more than a dozen methods for determining this important number. By making use of the Avogadro number we can, in principle, communicate our basic unit of mass, the gram, to an alien race. This is an important step since it changes the communication from the exchange of qualitative information to the exchange of quantitative information. Chemical reactions involve heat exchanges and so we may, assuming the gram has been understood, communicate our basic unit of heat, the calorie. Using that and the distinctive way in which pure substances melt and boil, we can communicate our temperature scale. Since at some point we would want to discuss the gas laws we should use the Kelvin scale; this is the scale that has degrees the same as those of the Celsius scale but starts at absolute zero; the latter is a real, experimentally verifiable, point on any temperature scale.

Chapter 14

The Special Theory of Relativity

Gene Roddenberry really started something when he created *Star Trek.* The idea of traveling around the galaxy with a heroic captain and an interesting crew—which included a few aliens—resonated deeply with a great many people. Part of the appeal may have been the vagabondish lifestyle this implied. Troubles, difficult attachments, maybe even financial responsibilities, were left behind as the spaceship traveled on to the next solar system. Part of the appeal also was the sense of anticipation as the crew entered a new, unknown, part of the galaxy. Who knew what they might find? This was partly the appeal of the Old West and the cowboy lifestyle. There was always another town to ride into and adventure to be had. The Old West is long gone, but Star Trek and the USS Enterprise beckon. Only now instead of a lonely ride between tiny centers of civilization, one had an interesting crew to interact with as you made your way to the next adventure.

But there is one fly in this ointment. It is called the "special theory of relativity." Star Trek is wholly dependent on "warp drive"—the engine that reduces some interstellar travel to days or weeks instead of years or decades, by enabling the crew to accelerate the space ship to speeds beyond that of light. Will we ever find such an engine? Many say, "Sure! "Why not?" or "Of course!" and yet there is always that niggling doubt stirred up by Albert Einstein.

Is he right or is this just another case of people saying something is impossible that later becomes commonplace? Weren't we told that man could never fly, or that the speed of sound was an unbreakable barrier beyond which airplanes could not go? So why is the "light barrier" any different? As has become my custom, I will present the facts as best I can and leave it to the reader to decide how to answer this question.

To begin with, there are two theories of relativity: special and general. Both are due to Einstein—the first in 1905, the second about ten years later. An immediate, and often overlooked, problem is the meaning of the word "theory."

The word means different things depending on the context in which it is used. In law, as I understand it, a theory of a case is simply an interpretation of the facts in that case—each side's story of how it unfolded. As one can easily imagine, in a criminal trial, even when the facts are not in dispute the prosecution and defense might have very different theories of the case. Suppose a police officer enters a convenience store late at night and finds the cash register open and a man with a handful of bills standing over the body of the clerk. If the man is arrested and his case comes to trial, the prosecution might claim that he killed the clerk and took the money. The defense, however, might claim that the man came to buy something, found the money on the doorstep and was returning it. Upon seeing the clerk on the floor he realized what had happened and was about to call 911, when the police arrived. These are two theories of the case. It is the jury's job to decide which of them is correct.

A theory can also mean a body of knowledge, and this is how the term is used in mathematics; this doesn't seem to be too well-known. Many seem to think that a theory in mathematics is like a theory in physics, but this is not so. A book on the theory of numbers, for example, simply contains a discussion of what we know about numbers as well as proofs that what the author says about these objects is in fact true. But the theory of relativity is a scientific theory and there (i.e., in science), the term has a very different meaning.

In the seventeenth century Robert Boyle showed that the volume of a gas, at a fixed temperature, was related to the pressure applied on that gas. This became known as Boyle's law (Ebbing 1987: 106–09). About a century later Jacques Charles showed that at constant pressure, volume and temperature were related. This, of course, became known as Charles's law. But why were these laws true? What was it about gases that made them work? An explanation—a "theory"—of why these laws hold true was developed by a number of scientists: James P. Joule in 1848, Rudolf Clausius in 1857, our old friend James Clerk Maxwell (Chapter 6) in 1859, and Ludwig Boltzman in the 1870s (Ebbing 1987: 127).

What they did was list a series of statements called postulates that, they assumed, described the gaseous state. These statements were based on observations and experiments. They then showed how the various laws could be deduced as logical consequences of the postulates.

They also deduced, from these postulates, other properties that gases should have. Experiments were then conducted to see if gases really did have these expected properties. If they did, then these experiments lend support for the theory. If they did not, then the theory would be called into question. It might have to be modified or even discarded. One can never prove a theory beyond all doubt. What if there are two or more theories proposed to explain the same set of phenomena? Then you accept, again and as always tentatively, the simplest, i.e., usually the one that makes the fewest assumptions.

Suppose, for example, that you see a spherical object high in the sky moving north at a steady rate of speed.

After five minutes, let's say, it goes behind a mountain and is lost from view. There are, at least, two theories one can come up with to explain what you've observed: (1) It is an alien spaceship traveling north because that's the direction the crew wants to go and, although you don't see any exhaust or vapor trail, the ship is probably powered in some way and can go in any direction its crew chooses; or (2) it is a balloon traveling north because that's the direction the wind is blowing.

Obviously the second theory is the simpler of the two. It is also testable. You can call the local weather station and find out if the prevailing wind at the time of your sighting was toward the north. Maybe the first theory is the true one, but there is no way to immediately test it, and so, until further evidence becomes available, one should be very hesitant to accept it; not because it is impossible for aliens to exist or for them to come here but because this observation can be explained in other, much simpler ways. The evidence does not support so extraordinary an interpretation.

The scientific study of motion began with some observations, and later some experiments, conducted by Galileo (Hawking 1988: 15–17). His first insight was that a material body had "inertia." We mentioned this in an earlier Remark (Chapter 9).

Galileo made another observation that is of interest here, one that those of us who travel by air are quite familiar with. He spoke of a ship moving linearly at constant speed, but we can think of an airplane whizzing along at a constant 600 miles per hour (in a straight line). Walking around requires no special effort, objects fall when they are dropped, liquids pour as expected; in other words, things behave as they would when the plane is at rest on the tarmac.

Albert Einstein gave a good deal of thought to what he called Galilean or inertial observers (Parker 1991). These were observers who were either at rest or moving at a constant speed in a straight line. If two such observers, in empty space, say, could communicate, they might disagree as to which of them is moving and which of them is at rest. Furthermore, unless they had some additional information, there is no way to settle the question. This, Einstein concluded, means that the motion of inertial observers is relative and not absolute. Either observer can claim that she is at rest and the other is moving and both would be right! Well, so what?

Einstein realized that this means that *the laws of physics must be the same for all inertial observers*; otherwise there'd be a way to tell which was at rest. This is the first postulate of special relativity.

But Einstein carried his thinking much further than this and came up with a second, much more far-reaching, postulate. This one requires just a little more discussion.

Let's suppose that you're in a car moving along a straight road at 30 miles per hour. Suppose that a housefly takes off from the back seat and flies to the dash board at a speed you measure to be five miles per hour. Now imagine that an observer sitting next to the road also measured the fly's speed. What would he get? He'd say the fly's speed was thirty-five miles per hour. Not really surprising since the fly started at thirty miles per hour, the speed of the car, and added his five miles per hour to that. But now suppose that someone in the back seat flashes a light beam forward. The speed of light is, for convenience, usually denoted by the letter c. So you, the driver, would measure the light ray as having speed c, but what would the observer on the ground, next to the road say? Shouldn't it be 30 + c, like the fly whose speed was 30 + 5? Einstein realized that this very common and seemingly natural answer was wrong!

No, despite how unreasonable it might appear, the car's driver and the man at the roadside would get the same value c when they measured the velocity of the light beam.

There was some experimental evidence for this conclusion. Ocean waves, of course, travel through water. Sound waves travel through the air. So what do electromagnetic waves travel through? It was conjectured that space was not empty but was filled with a substance called the "luminiferous ether." This explained many of the properties of light. But then the speed of light should vary with the direction that the earth was moving through this ether; the earth should have a wake through the ether like the wake of a boat through the ocean. Painstaking experiments, in particular one carried out by two American physicists, A. A. Michelson and E. W. Morley, showed that this was not the case. The velocity of light was independent of the direction it was going. So scientists abandoned the idea of the ether, and wondered what these experiments meant.

It was Einstein's insight that these results were pointing to a far more general conclusion, a conclusion that he took as the second postulate of special relativity: *The velocity of light is the same for all inertial observers.*

Only a genius could come up with postulate that is so remarkable, so counterintuitive, and so far-reaching.

Einstein went on to work out the consequences of his two postulates and these consequences are almost incredible. Perhaps the most counterintuitive deduction he made was that the measuring sticks of a moving observer must shrink, and his clocks must slow down in such a way that his measurement of the speed of light agrees with that of a stationary observer. There are mathematical formulas, called the Lorentz equations, that enable you to calculate just how much these sticks must shrink and how much the clocks must slow down. These weird effects have been demonstrated experimentally. They really do happen, and by the amount that Einstein calculated that they should happen.

Here is one test of the theory, and it passed it with flying colors.

What this means, since the kind of measuring device and the nature of the clock are irrelevant, is that space and time are not the disjointed, rigid entities postulated by classical physics. They are instead elastic and intimately connected. For the moving observer space shrinks and time slows down.

A second prediction of the theory is that the mass of a body is not constant. It increases as the speed of the body increases. This effect is negligible when the speeds involved are small compared with that of light. At such speeds the equations of Newton, which take mass to be constant, work just fine. A constant mass means that to increase a body's speed by a fixed amount requires a fixed amount of energy. Once that has been done, to increase the speed by the same fixed amount would require the same fixed amount of energy. But since mass increases each time you increase the speed by the same fixed amount, you need *more* energy, and the closer you get to the speed of light the more energy you need to accomplish the same increase in speed. To get even the smallest particle up to light speed would require an infinite amount of energy, and there is no source of this much energy. This too has been tested experimentally and it really does happen. Mass does increase with velocity and by the amount Einstein predicted that it should.

This is why many believe that the "warp drive" of Star Trek is a fantasy. I am always amazed at the reaction of some people when I present this consequence of relativity to a public audience. There are always a few who get extremely angry. They just don't want to hear this.

Another consequence of Einstein's postulates, perhaps the best known, is that matter and energy are equivalent. For a body with mass *m* this is expressed by the famous equation E, energy, equals m times c squared (c squared is just c times c). It is this equation that accounts for the terrible, yet awesome, power of the atom bomb.

The special theory of relativity, and all of its amazing predictions, has been tested extensively. It works! The expensive equipment used in the physics laboratory at Chern in Switzerland, and at Femilab in the United States was designed using the mathematics of special relativity. This equipment does exactly what it was designed to do. We cannot dismiss relativity as "just some theory" or "just Einstein's opinion," as I have heard some do. I repeat: many people do not want to hear this. The anger and even, believe it or not, the outrage I've heard expressed when I present these topics is quite remarkable and, I might be naïve, but quite unexpected. I don't like the limits relativity imposes on us either but, according to modern physics, that is the way that it is.

Relativity says that an ordinary particle can never be accelerated up to light speed. The photon (a particle of light) however, starts life at light speed. It doesn't have to be accelerated up to that speed. So why can't there be particles that, from their inception, travel at speeds faster than c? Such things have been discussed in the physics literature (Bilanuick et al. 1969).

They are called "tachyons." These, if they exist, would be pretty weird. Time, for them, would run "backward," i.e., opposite of how it runs for us, and giving energy to a tachyon would slow it down. Passing through the "light barrier" would be like Alice going through the looking glass. You enter a world where the bizarre becomes commonplace.

It would seem that the speed of light is a fundamental constant, a real limit on what one can hope to achieve in seeking an inter-stellar spaceship. Now, again, these conclusions are tentative. We may find a way around this limit someday, and maybe an alien race that is far older than we are has already found a way to surpass this limit. On the other hand, if inter-stellar travel must always take place at sub-light speeds, then such a journey must always be measured in years, and this applies both to us and to any aliens who choose to make the trip. If we are being visited by aliens, and according to some people this is happening often, then they are expending a great deal of energy to get here; yes, it is true that time would slow down for those on the spaceship, but they'd return home to find that those they left behind have aged far more than they have. This would be a problem for any star-traveling humans, but it might not matter much to a race of beings that live forever. Still they would miss a great many, perhaps important, events happening on their home planet. They'd have to have a pretty compelling reason to come here. I've heard it said that the reason they come here is that the human race is very amusing. They find us, and our antics, entertaining. So, to them, visiting us is like a trip to the zoo.

If we can't go to other star systems, at least we can communicate with their inhabitants via radio. Even that, however, will take years—radio waves travel at the speed of light but, as we have seen, the stars are light-years away. Maybe someday we'll find a quicker way to do this—perhaps we will discover some, as yet unknown, type of radiation that will enable us to exchange messages at speeds beyond that of light. At least then we might be able to engage in a dialogue. Special relativity has something to say about this as well. As we have, perhaps, come to expect, it is not what we might hope to hear.

Imagine an inertial observer, let's call her Ann, who sees two events P and Q. Let's suppose that she has a friend, another inertial observer we'll call Ben, with whom she can communicate, somehow, at faster than light speed. It turns out that if Ann sees event P first and then event Q and if she can tell Ben about them at faster than light speed, then we can locate Ben in such a way that he will see Q first and then P. Now before we ask the natural question, let us be clear. This is not like some stage magician sawing his shapely assistant in half. That is a trick, a clever illusion. Here P really happens before Q for Ann, and really happens after Q for Ben. Now for the natural question: So what?

Well, suppose I told you that the San Francisco earthquake of 1908 was caused by the atom bomb tests carried out in 1944. It seems plausible. The bombs certainly shook the ground so, maybe, they could touch off a quake. But it is clear that this can't be true. Nothing that took place in 1944 could be the cause of an event that happened in 1908. The cause must come *before* the effect. Now think back to our two events P and Q.

If P was the cause of Q, then P must happen before Q. P, for example, could be the event that Alice flips her light switch and Q could be the event that her light turns on. How then could Ben see Q followed by P? This is what people mean when they say that faster than light signaling violates causality.

Einstein has shown us that the universe is a pretty strange place. If Proxima Centurii (recall that this is the nearest star, four light-years away) were to explode tonight, we wouldn't see it for four years. That's how long it would take the light to reach us. But now we see that there is no way that we can know about this event until four years have passed. The event, although it has already happened, lies four years in our future. This too, the physicists tell us, is the way that it is.

REMARK: Space-Time, Higher Dimensional Spaces, and Hilbert Space

The space of our everyday experience seems to be three-dimensional. We can move left or right, backward or forward, and up or down.

We have no trouble visualizing this. Spaces of dimension two (planes), and dimension one (lines), are even easier to picture. But what about spaces of dimension greater than three? Is there any point in considering such things and, if so, what can we say about them?

The special theory led to the concept of "space-time," a four-dimensional entity. As we shall see (in the next chapter) the general theory requires this idea. The geometry of space-time, however, is non-Euclidean.

Here we shall focus on Euclidean spaces of arbitrary dimension.

An equation like $3x + 4y = 7$ is called a linear equation in two unknowns; x and y are the unknowns. We seek to replace x and y by a pair of numbers that make the equation true; here we could take x to be one, and y to be one, and the pair (1, 1) is called a solution to the equation. There are lots of other solutions; x could be 2 and y could be ¼, for example.

In many problems you have a pair of linear equations, each in two unknowns, and you look for a pair of numbers that satisfies both equations.

This is called a system of linear equations. Some systems have exactly one solution, some none, and some have infinitely many.

In the seventeenth century two Frenchmen, Descartes and Fermat, found that functions and equations could be represented geometrically; i.e., they could be graphed. This was a remarkable discovery that resulted in a new field of mathematics now known as analytic geometry. Here our powerful geometric intuition can be used to understand abstract problems of algebra, and the methods of algebra can be used to solve geometric problems.

The equation stated above is called linear because its graph is a line. Thus the behavior of a pair of linear equations becomes understandable. Two lines, in the same plane, can meet in exactly one point and when this happens the system has a unique solution. The two lines can also be parallel, such lines never meet, in which case the system has no solution. In the last case the two lines coincide (this can happen even when the equations look very different); here any pair of numbers that satisfies one of the equations also satisfies the other.

Some problems lead to linear equations with more than two unknowns, and sometimes to systems of such equations. A linear equation in two unknowns represents a line, but a linear equation in three unknowns represents a plane in three-dimensional space. The solutions of a system of these equations can be understood by visualizing how planes can intersect. A system of two equations each in three unknowns can have no solutions, a "single infinity" of solutions, or a "double infinity" of solutions. Again this becomes transparent when we consider the geometry.

The two planes can be parallel, like the floor and the ceiling of a room, in which case the system has no solutions. The planes can intersect in a line, like the wall and floor of a room. Every point on that line is a solution and so this situation was described as a single infinity of solutions. It can also happen that the two planes coincide, they are the same plane. Here we have a double infinity of solutions since every point on this plane satisfies the two equations.

In many problems one has to solve a system of linear equations that contain more than three unknowns. It was in trying to picture the possible solutions to such systems that mathematicians in the nineteenth century were led to investigate spaces of higher dimension. Each equation represents a flat surface in a space of dimension n, where n is the number of unknowns in the equation.

In two or three dimensions the geometry helps us understand the algebra, helps us see what the equations are telling us. It soon became clear, however, that in a higher dimensional space it was the algebra that helped us understand the geometry.

As we all know, each point in three-dimensional space is assigned three numbers, called its coordinates, (x, y, z). Given two such points, say (x_1, y_1, z_1) and (x_2, y_2, z_2), Euclidean geometry tells us that the distance between these points is given by the square root of the quantity $(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2$.

In an abstract space the geometry depends on how we *define* the distance between two points. In four-dimensional space, each point has four coordinates, and the distance between (x_1, y_1, z_1, w_1) and (x_2, y_2, z_2, w_2) is taken to be the square root of the quantity $(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2 + (w_1 - w_2)^2$. With this definition of distance the geometry of our space becomes Euclidean. The distance in n-space is defined in an analogous way and, for each *fixed* n, this too has the geometry of Euclid.

In this context the idea of dimension is rather straightforward. In a plane we can imagine two lines meeting at a point (such lines are said to be concurrent) and having an angle between them of 90 degrees; just look at the corner of this page. An angle of 90 degrees is called a right angle and two lines that meet at a right angle are said to be perpendicular. We can have two such lines in a plane, but we cannot have three concurrent lines in the plane each at right angles to the other two. This does happen, however, in three dimensional space.

The corner of a room shows three lines each one at right angles to the other two, but we cannot find four such lines. That only happens in a space of four dimensions. So we can think of the dimension of our space as the maximum number of mutually perpendicular lines we can have at any point.

David Hilbert (see Chapter 11) generalized these ideas to give us a type of infinite dimensional space that now bears his name. These were later applied to physics and now play a fundamental role in the foundations of quantum mechanics. In Hilbert space, each point has infinitely many coordinates, and finding the distance between two points involves an infinite sum. The coordinates are defined in such a way that the infinite sum has meaning. In a Hilbert space you can find infinitely many lines, each one perpendicular to all of the others.

The concept of dimension discussed here can be generalized so as to enable us to define the dimension of a space in which angle has no meaning. Such spaces arise in the study of trigonometric series (Chapter 7) and elsewhere in mathematics. We shall see that defining dimension in a more general setting turned out to be quite tricky and led to a new field of mathematics. There are several ways to do this and in some cases the same object can have two different dimensions. Such an object is called a fractal.

Would an alien race step from the algebraic solutions of systems of linear equations into the geometry of higher dimensional spaces, or would they be content to just solve the equations and not even consider their possible geometric meaning? Our approach enables us to use our considerable geometric intuition to further our understanding of the nature of these systems. If we contact an alien race that sees no need to do that, it might mean that they do not rely on the sense of sight to the same extent that we do. They might even be blind. We may have difficulty communicating with such a race.

The world at the atomic level is very different from the world of our everyday experience. The wave particle duality, the double slit experiment, and other counter-intuitive results challenged physicists for some time. Hilbert spaces provide a basis for the mathematics of the world at this level, and using them has proved fruitful. But it took the efforts of a number of brilliant physicists to come up with this approach. An alien race may find another way to deal with this aspect of reality.

Chapter 15

The General Theory of Relativity

Some years ago U Thant, one-time secretary general of the United Nations, had a conversation with J. Allen Hynek, chairman of the Astronomy Department at Northwestern University. Both men are now deceased. Mr. Thant said that, as a Buddhist, he believed in life elsewhere. He asked if Hynek thought that extraterrestrials might visit our world. Hynek replied that he too believed in life elsewhere, but that the times involved in journeys from outer space seemed insuperable. At that point the Secretary General made this remark: “Ah, but what may seem like years to us, may be just a day or two to others” (Hynek 1977: xiii).

Mr. Thant’s remark is thought-provoking. We like to think that there is an objective, physical, time. And we know that we experience time. But the two don’t always coincide. Time flies when we are having fun and drags when we are bored. Time hasn’t changed, only our experience of it.

Can there be beings whose perception of time is so radically different from ours that years seem like days to them? And if such beings exist, can we even hope to communicate with them? Maybe our life spans are, to them, so short that they would never even notice us. Let us look a little more closely at some of our ideas about these matters.

Time has been the subject of learned speculation for literally thousands of years, and yet the questions remain: What is time? Where did it come from? Where does it go?

And less seriously, "Why do we never have enough of it?" The first of these is, of course, not the kind of question one expects to answer. After all, we don't know what anything really "is." We would ask this question only to start a discussion, a discussion that might give insight into the nature of this elusive entity that we call time. There seems to be no doubt that all humans experience a phenomenon that we call the "passage of time." This personal experience is probably more convincing of the reality of time than any definition could be.

It is, however, very difficult to pin down the nature of "temporal experience" (Whitrow 1961). So, rather than trying to define time, let us agree that we know what time is, as long as no one asks us to explain it. Saint Augustine, who wrote extensively about time, said pretty much the same thing. As he put it: "If no one asks me, I know; but if any person should require me to tell him, I cannot."

The question of where time "came from," where time originated, has been discussed by modern cosmologists. The current view is that when the universe began, in the Big Bang, both space and time were created. Even they quote Saint Augustine: "The world was created with time and not in time." Hence it is meaningless to ask what occurred "before" the Big Bang because "before" is a temporal term and to speak of happenings before the Big Bang is to put this event *in* time. (I should mention that some physicists are proposing a theory, M-theory, that does suggest that time existed "before" the Big Bang. To my knowledge, however, this is still under discussion.)

These last two paragraphs illustrate one of the most insidious difficulties one encounters in discussing time. It is almost impossible to avoid using words that have temporal connotations—words like "before," "later," "now," and even "modern" or "current." These words are so common that we assume, usually tacitly, that their meanings are clear. This assumption is, at best, questionable. In fact, the philosopher Hans Reichenbach once said that the problem of time is largely one of "explication" (Reichenbach 1957); i.e., we must gradually replace the vague or ill-defined words we commonly use to discuss time by more precisely defined terms. No easy task.

The question of where time "goes" and the more facetious one of why we never have enough of it allude to the persistent feeling we have that time "flows," or that there is a motion from the past into the future, an ever fleeting "present moment." But this strong, one might say defining, aspect of psychological time has no counterpart in physics. According to one writer:

> Present day physics makes no provision whatever for a flowing time, or for a moving present moment.
>
> Those who might wish to retain these concepts are obliged to propose that the mind itself participates in a novel way in some form of physical activity that is not manifest in the laboratory, a suggestion that meets with a great deal of reserve among the scientific community.
>
> Eddington [Sir Arthur Eddington, a distinguished British physicist] has written that the acquisition of information about time occurs at two levels: Through our sense organs in a fashion consistent with laboratory physics, and in addition through the "back door" of our minds. It is from the latter source that we derive the customary notion that time "moves." (Davies 1974)

We might mention that the great German physicist Hermann Weyl expressed, somewhat more succinctly, the same view when he said, "The real world doesn't happen it just is." To understand this rather static view of reality we have to recall something about the special theory of relativity (Chapter 14).

Imagine two observers, one at rest and the other moving at constant speed in a straight line. All clocks on the moving system would appear, to the stationary observer, to have slowed down.

This effect is independent of the particular nature of the clock. A digital watch, an old-fashioned pendulum clock, etc., would all slow down and by the same rate. Thus, to the stationary observer, time itself has slowed down on the moving system. Furthermore, measuring rods, whatever their composition, will shrink in the moving system—it is this shrinkage of measuring rods and slowing down of clocks that "causes" the moving observer to come up with the same number that the stationary observer will find when they both measure the velocity of light. Thus space and time are not the disjointed, absolute, rigid entities postulated by classical physics. They are instead elastic and interconnected.

This led Minkowski, one of Einstein's teachers, to introduce a new, combined entity he called "space-time." The points of space-time are called *events* and Minkowski gave us a way to measure the "distance" between two events. In this way he gave space-time a geometric structure. The growth of a plant, say, is not regarded as an unfolding through time, but as a set of points in space-time, as a "world-line."

The only aspect of this picture that some find uncomfortable is the fact that space-time is four dimensional; it is often referred to as the Minkowski 4-world.

Our view of both space and time is a little paradoxical. We say a line is one dimensional but it is made up of dimension-less points (see the Remark below), and we say time consists of instants which have zero duration. But the clock reads noon for just an instant, so it reads noon for no time at all! The early pioneers in the study of motion realized that they had to consider instantaneous velocities and accelerations. This means we close down on a point with shorter and shorter intervals and divide the lengths of these intervals by shorter and shorter intervals of time. Thus time and space are regarded as infinitely divisible. It seems that this view of time is a relatively modern one.

As we have already noted (Chapter 9), in the year 470 CE the Roman philosopher Martianus Capella suggested that there might be "atoms" of time. This hypothetical entity is sometimes discussed by modern physicists who call it a "chronon."

The difference between these two models is not trivial. Chronones, if they exist, would be lined up like the beads on a string, each having a unique predecessor and a unique successor. Removing a chronon would leave a gap in the time line, a gap with well defined edges and duration. By contrast the instants are lined up but not at all like the beads on a string. They are arranged like the real numbers, the mathematical continuum (Chapter 11). Between any two instants there are infinitely many other instants. Given one there is no "next" one, and removing an instant leaves a gap in the time line, but it is a gap with no edges and zero duration. This modern view of time, so crucial for the science of physics, may have arisen with the development of music and the enormous popularity of music may have caused this view to replace the older one.

In the general theory of relativity Einstein gave us a new way to understand gravity. The classical picture says that the earth orbits the sun because their mutual gravitational attraction produces a force that compels the earth to do so. In this new view the sun distorts the geometry of space-time, causing it to curve. The earth orbits the sun because it is following the contours of space-time in the solar neighborhood. The physicist John Wheeler expressed it like this: *mass tells space-time how to curve, and curved space-time tells mass how to move* (Webb 2002: 68).

The general theory also led to the idea of a black hole. A star keeps itself from collapsing by burning its hydrogen. Eventually, it runs out of this (I'm simplifying a great deal here) and shrinks. Depending on the initial mass of the star it can shrink to a white dwarf, a neutron star, or a black hole. A white dwarf, and these have been observed, has a radius of a few thousand miles (by contrast the sun, an average star, has a radius of over five hundred thousand miles) and is so dense that a cubic inch of it would weigh hundreds of tons (Hawking 1993: 118–19).

In 1967 a student at Cambridge, Jocelyn Bell, discovered some objects that were emitting regular pulses of radio waves. Both she and her supervisor, Anthony Hewish, thought that they might have discovered an alien civilization. They named the first four sources LGM one to four, the LGM meaning "Little Green Men." Unfortunately, the objects, now called pulsars, turned out to be rotating neutron stars. Such stars have a radius that measures only tens of miles and are even denser than white dwarfs. A cubic inch of a neutron star would weigh millions of tons (Hawking 1993: 119).

A neutron star has an extremely powerful magnetic field (see the Remark in Chapter 4). This is measured in units called the gauss (named for the great German mathematician Karl F. Gauss). To get an idea of the power of the field around a neutron star we might note that the magnetic field of the Sun is about 2 G, and that of some white dwarfs is about 1,000,000 G, or 10^6 G. The field around a neutron star, however, can be as high as 1,000,000,000,000 G, or 10^{12} G!

A rotating neutron star will radiate energy into space somewhat like a lighthouse. The pulse is along the magnetic axis which is not the same as the axis of rotation. So the radiation sweeps out a cone and, if the earth happens to be on that cone, a brief flash will be observed as the beam of radiation passes across our line of sight. This is the cause of the regular pulses detected by Dr. Bell.

The term black hole was coined by the American physicist John Wheeler in 1967. A black hole is the result of a star collapsing into a smaller and smaller region, causing its gravitational field to increase in strength. Eventually this field becomes so strong that even light cannot escape.

According to Stephen Hawking, "The laws of physics are time-symmetric. So if there are objects called black holes into which things can fall but not get out, there ought to be other objects that things can come out of but not fall into. One could call theses white holes. One might speculate that one could jump into a black hole in one place and come out of a white hole in another" (Hawking 1993: 119).

It has been suggested that a spaceship could enter a black hole and come out of a corresponding white hole—the connection between the two was referred to as a wormhole. In this way one might be able to circumvent the light barrier; i.e., one could enter a black hole in one portion of the galaxy and emerge soon thereafter in an entirely different portion of the galaxy.

The equations of general relativity, the Einstein field equations, led to the idea of a black hole long before such things were discovered. Unfortunately those same equations tell us that wormholes, if they exist, would be very unstable. A spaceship entering one would cause the destruction of both the ship and the wormhole (Hawking 1993: 119–20).

The view that time and space are continuous breaks down at the atomic level due to quantum effects. There is a length below which the general theory breaks down, the so-called Planck length (Kaufmann 1994: 531). The time it takes light to traverse this length is called the Planck time. It is extremely small and it is sometimes suggested that this might be the duration of a chronon.

REMARK: The Geometry of Minkowski's 4-World, and Why Points Are Zero Dimensional

The general theory of relativity is concerned with how massive bodies curve space-time. Thus this "hybrid" entity is viewed as the fundamental fabric of the universe.

The geometry of Minkowski's 4-world is non-Euclidean. Each point in this space, called an event, has four coordinates; three representing the event's location in space and one representing the time at which the event occurred. The distance between two events, say (x_1, y_1, z_1, t_1) and (x_2, y_2, z_2, t_2), is defined to be the square root of the quantity (compare with the definition of distance in Chapter 14) $(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2 - (t_1 - t_2)^2$.

The presence of the minus sign before the last term brings space-time into conformity with the facts of special relativity and shows that the geometry is non-Euclidean.

It is possible, because of the minus sign, for this quantity to be zero. The points for which this is true describe a cone in space-time, called the light cone. The lines on that cone have zero length and are interpreted as the paths of light rays.

The geometry of space, on the scale at which it is studied by cosmologists, is more complicated than indicated above. Space-time is curved in the vicinity of large masses. The geometric tools needed to deal with the problems of cosmology are quite sophisticated (Adler, Bazin, and Schiffer 1965).

Euclid defined a point as "that which has no part." We usually say a point has dimension zero, or that a point has length zero. The question is sometimes asked, "Why can't a point be like a tiny disk, say like the period at the end of a sentence, and have a small, but positive, diameter d?" If one is willing to agree that all points must then have the same diameter, then you get into a contradiction. To see this suppose that l_1 and l_2 are any two line segments. Then l_1 must have length md, where m is a whole number, and l_2 must have length nd, where n is also a whole number.

It follows that the length of l_1 divided by the length of l_2 must be m/n. In other words, assuming that points have positive diameter implies that the ratio of the lengths of any two line segments is a rational number.

In particular, if we look at a right triangle (one containing a 90-degree angle) whose legs are of length 1, then the length of the hypotenuse (the side opposite the 90-degree angle) divided by 1 must be a rational number, because it is the ratio of the lengths of two line segments. But from elementary geometry (for a right triangle with legs of length a and b, the hypotenuse can be found from the equation $a^2 + b^2 = c^2$. This is the famous Pythagorean Theorem. When both legs have length 1, then $c^2 = 2$) we know that the length of the hypotenuse is $\sqrt{2}$.

Let us show that $\sqrt{2}$ is not a rational number. This is the proof that, allegedly, cost an ancient Greek sailor his life (Chapter 4). We argue by contradiction. Suppose that $\sqrt{2} = m/n$ where m and n are whole numbers. If these numbers have any common factors we can cancel them out, so we may suppose that m and n have no common factors. This is a crucial observation.

Now we square both sides of our equation and get $2n^2 = m^2$. In other words, m^2 is an even number. But when you square an odd number you get an odd number. So if m^2 is even, then m must be even. This means that m is twice something, say $m = 2p$. Then $m^2 = 4p^2$. Putting our equations together we get $2n^2 = m^2 = 4p^2$, or $n^2 = 2p^2$. From this we see that n^2 is even and so n must be even. We have reached a contradiction because m and n are both even which means they have a common factor of 2.

Chapter 16

The University of Colorado Study

Scientists rarely investigate UFOs. This is not at all surprising. The typical scientist spends many years learning difficult, highly technical material. He or she must then learn how to conduct research. This is done in a more specialized area that is of particular interest to the person involved. Scientists who go into academic life face stiff competition for relatively few jobs, and, if they do get a job, they are expected to produce high-level research. They are under pressure to publish and under even greater pressure to attract money to support their research.

Now before any agency awards a grant of several thousands and in some cases several millions (a friend of mine, a cancer researcher, received a grant of five million dollars) of dollars to an individual, the agency wants some assurance that that individual is competent, of course, but also that the person is reliable and level-headed.

Needless to say, anyone involved in this kind of high-pressure career is busy—there are always more problems to investigate in any area of science, and I mean fascinating, challenging problems of manifest importance. One would hardly expect the average scientist to drop everything and go chasing UFO stories.

A scientist must also be careful to maintain his or her reputation. Involvement with anything "fringy" like UFOs can cause irreparable damage to one's credibility; that scientist won't be taken seriously by colleagues or by those who serve on the committees that award grants. Maybe it shouldn't be this way, but this is the way that it is.

So how is it that the distinguished physicist Edward U. Condon agreed to lead a team of scientists in a two-year investigation of the UFO problem? And how is it that he got half a million dollars from the government to do it? The short answer is simple: Swamp gas! (Jacobs 1975: 201; see also Peebles 1994: 169–90).

The report of the Condon committee, as it came to be called, enabled the Air Force to terminate, probably with a sigh of relief, its twenty-two-year involvement with UFOs. In December of 1969 the secretary of the Air Force, Robert C. Seamans, officially terminated Project Blue Book, ending, although some don't really believe it, all Air Force interest in the UFO question. This had begun as "Project Sign" in September of 1947. That was changed to "Project Grudge" on 11 February 1949 and this, in turn, became Project Blue Book in the summer of 1951. Those involved in the project were located at Wright-Patterson Air Force Base in Dayton, Ohio.

It was realized early on that an astronomer was needed to advise the Air Force investigators. Many reports were, to an astronomer, obviously meteors, twinkling stars, and, quite often, the planet Venus. A natural choice for this role was the director of the nearby McMillan Observatory at Ohio State University. That was Dr. J. Allen Hynek. His association with the Air Force continued even when he left Ohio State and became the director of Northwestern University's Dearborn Observatory in Evanston, Illinois.

In 1966 there were a number of UFO sightings in the state of Michigan, some of them quite spectacular—sightings of long duration, involving many independent witnesses (Peebles 1994: 169–70). Dr. Hynek, perhaps to his regret, traveled to Michigan to investigate. Now, unfortunately, a few of these sightings were of faint, flickering lights seen over swampy areas. These, and only these, have a rather simple explanation. Rotting vegetation present in swamps will produce gases, some of which, when they leave the water, will spontaneously combust. This phenomenon is well-known in certain areas and is sometimes referred to as a "will-o'-the-wisp" or "foxfire."

After consulting with colleagues at the local university, Hynek gave this explanation to the media (Jacobs 1975: 301; see also Peebles 1994: 170–72). It was a mistake. He was accused of saying that the entire UFO phenomenon was caused by swamp gas! The cartoonists of the time had a field day, all at Hynek's expense. In one of these, for example, we see the three wise men gazing at the star of Bethlehem; one of them turns to the other two and says "swamp gas."

Those involved in the more puzzling sightings were not amused. To them it seemed that the Air Force was ridiculing their reports and dismissing what they saw with a silly, flippant explanation. Their complaints reached the ears of two of Michigan's congressmen, Weston E. Vivian and Gerald R. Ford. Eventually congressional hearings were held (5 April 1966) and money was allocated for an independent (i.e., non-Air Force) study of the UFO question. A number of universities declined the grant, but it was eventually accepted by the University of Colorado at Boulder. The initial grant was for $300,000 plus $13,000 to cover overhead, but this was raised to a final total of $525,905 (Peebles 1994: 174; see also Jacobs 1975: ch. 9). On 7 October 1966, the Air Force announced that the University of Colorado had been selected to conduct a study of UFOs. Dr. Edward U. Condon would be the project director and Assistant Dean Robert Low would be the project coordinator. Dr. Franklyn E. Roach, an astrophysicist with the Environmental Services Administration, and Dr. Stuart W. Cook, chairman of the psychology department, were to be the principal investigators. Other staff members included Drs. Saunders, a psychologist; Levine, an electrical engineer; and Craig, a physical chemist.

I should mention that Dr. Hynek welcomed this development and felt a small sense of personal vindication. He had, for years, been calling for a serious, scientific investigation of the many very puzzling reports that had come to his attention, reports by competent witnesses that couldn't be easily explained (Hynek 1966: 205–19). He was not, however, at all happy with Dr. Condon's conclusion (see below).

The project was plagued with problems almost before it began. In a lengthy memo urging the University to take the grant, Assistant Dean Robert Low wrote:

> Our study would be conducted almost exclusively by nonbelievers who, although they couldn't possibly prove a negative result, could and probably would add an impressive body of evidence that there is no reality to the observations. The trick would be, I think, to describe the project so that, to the public, it would appear a totally objective study but, to the scientific community, would present the image of a group of nonbelievers trying their best to be objective but having an almost zero expectation of finding a saucer. One way to do this would be to stress investigation, not of the physical phenomena, rather of the people who do the observing—the psychology and sociology of the persons and groups who report seeing UFOs. If the emphasis were put here, rather than on examination of the old question of the physical reality of the saucers, I think the scientific community would quickly get the message. (Peebles 1994: 175–76)

This memo, which has come to be called the "trick" memo, was filed and forgotten for a time, but later rediscovered with disastrous results.

There was also the alleged negative attitude of the director. In a speech given less than three months into the project Edward Condon said: "It is my inclination right now to recommend that the government get out of this business. My attitude right now is that there is nothing to it." He is said to have added, with a smile, "but I'm not supposed to reach that conclusion for another year" (Peebles 1994: 180). It is also alleged that he spent an inordinate amount of time on the antics of the "lunatic fringe." For example, he is said to have passed on to Washington, with a straight face, an offer made to him by "an agent of the Third Universe" to construct, for three billion dollars, a space port so that spaceships from this universe could land on our world (Hynek 1972: 234).

At some point Dr. Roy Craig, while preparing for a speaking appearance, rediscovered the trick memo. He showed it to the other members of the committee with the comment: "See if this doesn't give you a funny feeling in the stomach" (Peebles 1994: 181). Low's behavior also upset some of the staff. He took a month-long trip to attend the International Astronomical Union meeting in Czechoslovakia. Saunders assumed he would take the opportunity to meet with the editor of *Flying Saucer Review* and also to meet with the well-known French UFO writer Amie Michel. Instead he went to Loch Ness, leading the staff to feel that he was equating UFOs with the Loch Ness Monster (Peebles 1994: 180).

The trick memo was shown to colleagues outside the committee by Saunders and Levine. When Condon heard of this he promptly fired the two men. This led to further morale problems within the group and, two weeks later, Mary Louise Armstrong, Condon's administrative assistant, resigned from the project (Hynek 1972: 239). Her illuminating letter of resignation can be found in appendix 3 of Hynek's book.

These were the internal problems. There were also external problems. On 14 May 1968 an article titled "Flying Saucer Fiasco," written by John G. Fuller, which centered on the trick memo, appeared in *Look* magazine. It depicted the Condon study as being biased against UFOs from the start due to the alleged prejudice of both Condon and Low (Peebles 1994: 183).

And there was more. Congressman J. Edward Roush (D-Ind.) attacked the study on the House floor: "The story in *Look* magazine raises grave doubts as to the scientific profundity and objectivity of the project conducted at the University of Colorado," he said. "The publication of this article will cast in doubt the results of that project in the minds of the American public; in the minds of the scientific community. We are poorer—$500,000 later—not richer in information about UFOs. Where do we go from here? I am not satisfied; the American public will not be satisfied" (Peebles 1994: 183).

The Condon committee examined 91 cases of UFO sightings, some visual sightings by multiple witnesses, some involving photographic evidence, some involving both visual and radar evidence. Of these, 30 were listed in the final report as "unidentified." This seems significant; the investigators were highly competent, well-respected scientists. Yet Dr. Condon's summary of his committee's work was quite negative (Peebles 1994: 187):

> Our general conclusion is that nothing has come from the study of UFOs in the past 21 years that has added to scientific knowledge. Careful consideration of the record as it is available to us leads us to conclude that further extensive study of UFOs probably cannot be justified in the expectation that science will be advanced thereby.

The staff members' individual conclusions were not nearly so negative. Why this discrepancy? An interesting explanation was suggested by Stanford University Professor Peter A. Sturrock. His remarks about quasars are surely dated, but still the statement is worth quoting:

> The evaluation of the evidence by category, presented in Section IV, seems to show that each staff summary is a fair and justifiably cautious summary of the relevant case material. By contrast, Condon's summary seems not to represent an accurate reduction of the observational data. Hence the basic weakness of the Project is that the efforts of many individuals found no satisfactory integration.
>
> This failing may have been due in part to a faulty initial conception of the nature of the phenomenon. If, as the Director may have believed, the phenomenon could be tackled as a straight-forward problem of physical science, there might now be little disagreement among the scientific community regarding the validity and conclusions of the Report. The UFO phenomenon appears instead to be more akin to some of the enigmatic phenomena of modern astronomy, such as quasars. Concerning these strange objects, we are not at all sure where they are, we are even less sure of what they are, and we have very little idea of how they function. Concerning UFOs, we are not sure whether they are hoaxes, illusions, or real. If real, we do not know whether the reality is of a psychological and sociological nature, or one which belongs to the realm of physics. If the phenomenon has physical reality, we do not know whether it can be understood in terms of present-day physics, or whether (like quasars) they may present us with an example of twenty-first century (or thirtieth century) physics in action. If one is indeed facing a problem of this magnitude, it is necessary to devote the utmost care to the scientific methodology involved in the project.
>
> In sum it is my opinion that the weaknesses of the Condon Report are an understandable but regrettable consequence of a misapprehension concerning the nature and subtlety of the phenomenon (Sturrock 1974: 27–28).

The UFO mystery, in its modern form, has been with us for more than sixty years now, and we seem to be no closer to a solution than we were back in 1947. Sightings continue (e.g., the Phoenix lights [Kitei 2010]) despite the ridicule, the wisecracks, the subtle and not so subtle attacks on the credibility, and sometimes even the mental health, of those who report such things. But Dr. Hynek, who interviewed hundreds of witnesses in his twenty-two year association with the Air Force, claims that most were competent, reliable people from all walks of life (Hynek 1972: 7–24). Of the Condon committee, he says:

> The … major mistake made by the Condon committee was to consider only the problem of whether UFO reports … supported the hypothesis that the earth was being visited by extra-terrestrial intelligences. But the real problem was—and remains—whether UFOs are something genuinely new to science, quite apart from any preconceived theory to account for the reports. We need to consider the UFOs without preconceived hypotheses.… The solution of the UFO phenomenon … may not be easy to accept. It might well call for a rearrangement of many of our established concepts of the physical world that will be far greater even than the rearrangements necessary when relativity and quantum mechanics entered our cozy little world (quoted in Steiger and White 1973: 68).

Will the mystery simply fade away? After more than sixty years, it doesn't seem like it. Is its longevity due to the fact that it fulfills some deep psychological need, or is there, hidden in all the bewildering data, a real, unknown phenomenon? At this point in time, no one really seems to know. But note how in the statements of Sturrock and Hynek, they don't speak of spaceships or alien visitors, just some possibly new phenomenon. It seems clear, at least to me, that UFOs and SETI are totally unrelated subjects, and if we are ever going to detect aliens, it will be through the efforts of SETI scientists because UFOs, whatever they are, are not space ships piloted by alien beings!

REMARK: Space as Multi-Dimensional, the Dimension of Sets, and General Topology and Functional Analysis

It has been suggested that UFOs come from another dimension. This would explain how they could appear and disappear at will. We have seen how higher dimensional spaces were introduced into mathematics in the nineteenth century, in connection with linear equations (Chapter 14). So the idea is that we live in a universe that has many spatial dimensions, perhaps infinitely many, but we are aware of only three of them. As we have seen it is easy to define the dimension of the spaces introduced in Chapter 14 in a way that is intuitively satisfying. Not surprisingly, Euclidean n-space has dimension n.

In the late nineteenth century certain infinite dimensional spaces made their way into mathematics. Progress in the study of these objects was slow until mathematicians developed more sophisticated tools. The necessary ideas arose from a more penetrating study of space that was carried out in the twentieth century. The study began with early attempts to define the dimension of certain subsets of the plane. Two results found in the late nineteenth century brought this problem to the attention of the mathematical community.

Georg Cantor, in his investigations into infinite sets, was able to set up a one-to-one correspondence between the points of a line segment, of length 1, say, and the points in the unit square (a square having each side of length 1). This seemed "wrong" somehow. The line segment is one dimensional, while the square is two dimensional, so how could they be in one-to-one correspondence? But, if nothing else, Cantor's work made it clear that infinite sets had weird properties, so no one took this too seriously.

That attitude changed when Guiseppi Peano constructed a *continuous* function from the line segment onto the square. A continuous function maps points on the segment that are close together onto points of the square that are close together. Unlike the somewhat strange functions one encounters in parts of set theory, continuous functions are at the heart of calculus and other areas of analysis.

Peano's disturbing result led many mathematicians to undertake a penetrating analysis of the properties of subsets of the line and plane, and to undertake a deep study of the properties of continuous functions. Also, around 1900 a new theory of integration was introduced by the French mathematician Lebesgue. His work had a profound effect on mathematics and involved assigning a measure to subsets of the line (Hewitt and Stromberg 1969). The result of these developments was the creation of a new branch of mathematics called "general topology." Groups of mathematicians in Poland, in Germany, in The Netherlands, in France, in the United States, and in Texas (yes, Texas had its own group of topologists) investigated this new area. Early in the twentieth century this caused some problems. Communications were much slower, and each group came up with its own terminology. Sometimes the same concept had different names in the different countries (Folland 2010) or in the same country; I've been told that, in the early days, American topologists and Texas topologists had trouble understanding each other.

In general topology a detailed study is made of the meaning of "closeness" of points to sets, and of the possible connections between a continuous function and the properties of the set of points on which it is defined. General topology has many applications to many areas of mathematics including real and complex function theory, modern differential geometry, and differential equations.

It was in the late nineteenth and early twentieth centuries that certain infinite dimensional spaces arose in connection with applying the Lebesgue integral to the investigation of trigonometric series, and in connection with other problems. Some of the early investigators tried to treat them as simple extensions of the finite dimensional case, but this approach, which is partially successful in the case of a Hilbert space, did not lead very far in the cases of interest here (DeVito 1978). By bringing in the ideas of general topology, however, the study of infinite dimensional spaces blossomed into a vast area now known as functional analysis. This was the subject of an enormous amount of research in the twentieth century that continues today.

Functional analysis, as abstract as it might seem from the description above, has a kind of "human" aspect. Mention of the set of continuous functions on the line results in a shrug. But call it the space of continuous functions and you find people asking questions: Can you actually measure distance in that space? What is a sphere like in such a space? Is there anything like a plane in that space? Answering these questions leads to some deep mathematics, but the questions didn't even arise until we called the object we started with a "space" instead of just a "set." Even in this extremely abstract setting, our powerful geometric—shall I say "visual"— intuition guides us.

Chapter 17

Surprise!

A number of developments in the late twentieth century gave strong support for the idea that life can exist on other worlds, and that it can arise much more easily than previously thought. The first of these was a shocking discovery about life right here on Earth.

In 1977 the submersible *Alvin* discovered areas on the ocean floor where lava was rising from the Earth's core. This didn't surprise the geologists, many of whom had predicted that such regions should exist. It was the biologists who were in for a real shock. At a depth where the pressure is enormous, and sunlight never reaches, there were hundreds of living organisms. Furthermore, 95 percent of these were new to science. Their source of energy was the heat gushing out from the nearby vent. This discovery led to new ideas about the origin of life and a revision of many long-held beliefs about where life might be found.

Soon, life was being found in many places once thought impossible. Highly acidic pools, for example, and pools with a high alkaline or salt content. Organisms have also been found deep underground and in sulfurous springs (Darling 2001: 21–25; Kaufman 2011: 15–34).

These discoveries, together with the rapidity with which life developed on Earth (as shown by the fossil record) has led many scientists to believe that life will arise wherever conditions are favorable. The case for extraterrestrial life seems much stronger. These facts also lead to two intriguing questions: Might life have arisen more than once, and might it be happening now? As far as I know, no evidence for an affirmative answer to either question has been found to date.

Another development that seems to imply that life may arise easily took place in pure mathematics. It was found that very simple processes can, after a few repetitions, result in surprisingly complex structures. A case in point is function iteration. Given a function, we may apply it to a number to obtain a new number. Applying the function to this new number results in another number, and we may apply the function to that. This process is called function iteration and, even in the case of simple functions, leads to remarkably complex structures. This is best seen by setting up computer models of the process and graphing the results (Crownover 1995).

Another striking case where complex structure arises from simple beginnings is that of cellular automata. Imagine an infinite chess board. Each square can be in one of a finite number of "states" which can be visualized as colors. Starting with simple rules for changing states, like requiring that any yellow square that touches three blue squares must turn red, very complicated behavior is observed after a few iterations.

These are examples of what are called complex systems. I quote Ian Stewart (2007: 276):

> Complex systems support the view that on a lifeless planet with sufficiently complex chemistry, life is likely to arise spontaneously and to organize itself into ever more complex and sophisticated forms.
>
> What remains to be understood is what kinds of rule lead to the spontaneous emergence of self-replicating configurations in our own universe—in short, what kind of physical laws make this first crucial step towards life not only possible, but inevitable.

REMARK: Fibonacci Numbers and the Golden Ratio, Logarithms, Exponentials, and the Number e, Connections to the Complex Numbers

Leonardo of Pisa, also known as Fibonacci, investigated many arithmetic problems. Among them was this: Suppose that a pair of rabbits takes two months to mature and, after that, produces a new pair a month, how many pairs will there be each month?

So in the first month we have one pair, and in the second month we also have one pair. In the third month however we have two pairs, the original pair and the one they, having reached maturity, produced. Continuing the counting we arrive at the sequence 1, 1, 2, 3, 5, 8, 13, 21, 34 . . . . Each number, after the first two, is the sum of the two preceding it: 2 is 1 + 1; 3 is 1 + 2; 5 is 2 + 3, etc.

These are the Fibonacci numbers, and they are found in a great many biological contexts; the way leaves grow on various trees, the geometry of certain sea shells, etc. It has been suggested that these numbers would be known to an alien race because they show up in so many diverse areas. Furthermore, the quotients of these numbers—1/1, 2/1, 3/2, 5/3, 8/5, 13/8, etc.—get closer and closer to an irrational number that some think plays a role in aesthetics. This is the "golden mean" or the "golden section" or "golden ratio," This number also appears in many biological contexts; in the human lung for instance (Lemarchand and Lomberg 2011: 384–85).

The ancient Greeks found this number by solving the following problem: Suppose that we have a line segment consisting of two parts, a larger part we'll call l, and a smaller we'll call s. The total length of the segment is then l + s. Suppose that the total length is to the larger part as the larger part is to the smaller; i.e., suppose that (l + s) / l =

l / s. What is the number l / s? If we solve this equation for l / s we get $(1 + \sqrt{5}) / 2$ which works out to be about 1.618.

Some say that a person's navel divides his or her body in this "golden" ratio. There have never been any studies done on this, at least none that I am aware of. I once mentioned this to a friend, a social scientist, and asked why no one had investigated this matter. She looked at me strangely and said that she didn't think the question was of overriding social significance.

The number systems of many early societies were very crude. Many simply used their alphabet to designate the first few natural numbers. This made calculation extremely difficult. Fibonacci was instrumental in bringing the Indian-Islamic system of enumeration to Europe. A great advantage of using that system of enumeration, besides its economy of symbols, is the ease with which one can do arithmetic. As time went on, however, the calculations involved in certain problems, problems in astronomy for instance, became prohibitive. A tremendous breakthrough came in the sixteenth century when the Scottish mathematician J. Napier and, independently, the Swiss mathematician J. Burgi, introduced the idea of a logarithm (Richardson 1950: 233).

Nowadays these functions play a role in both pure and applied mathematics that has little to do with calculation. This is true of the trigonometric functions as well. It should be understood, however, that the original reason for interest in these functions was due to their use in doing practical calculations.

For each number b, greater than 1, there is a function called the logarithm base b. This is usually denoted by $\log_b$ or, when the base is understood, simply by log. In each case the domain of the function is the set of positive real numbers, and the co-domain is the set of all real numbers (Chapter 6). These functions have some nice properties. They are one-to-one, which means they can be "undone," and the logarithm of a product is the sum of the logarithms of the two factors. So if A and B are two numbers we can find the log(AB) by finding logA and logB and adding these numbers. Now that we know log(AB), we just "undo" it to get AB; so instead of multiplying two large numbers A and B, we just add their logarithms. Similarly, division becomes a problem in subtraction.

Since our number system is based on ten it is simplest, for calculating, to use ten as the base of our logarithms. These are called "common" logarithms.

Since the logarithm is one-to-one there is a function that undoes it; it has an inverse. The inverse of the logarithm is the exponential function. Let us work with 10. The function that takes a number N to the number 10^N is called the exponential function (base 10). The common logarithm, $\log_{10}$, undoes this function; i.e., it takes 10^N back to N. As we have seen the function that takes N to the number $\log_{10}N$ is called the log function. The exponential function undoes this; it takes $\log_{10}N$ back to N. Thus, 10 to the power $\log_{10}N$ is just N.

With the development of calculus it was found that differentiating certain functions at each point of their domains gave us another function. This is true of any logarithm, but the formulas become especially simple when we use as our base the number e = 2.718281828459045 . . . . It is an irrational number and never ends.

It turns out that this number, and any power of it, can be easily calculated from an infinite series. The logarithm base e is called the "natural" logarithm and is often denoted by ln. The inverse of ln is the function e^x. This function is found virtually everywhere in pure and applied mathematics. It is so useful, and so easy to calculate, that this might be a good candidate for inter-stellar communication.

The calculations discussed here are now hidden within our hand calculators and computers, and we are rarely aware of them. Infinite series, however, are often the only way to calculate the values of many important functions like the sine and cosine discussed in chapter seven. It was by using infinite series that the Swiss mathematician Euler was led to define e^{ix} to be (cos x + isin x); here $i^2 = -1$, and x is real. This extremely useful formula has an amusing consequence.

We have $e^{i\pi} = \cos \pi + i\sin \pi = -1$. This may be written $e^{i\pi} + 1 = 0$, a formula that contains the e of calculus, the i of algebra, the π of geometry, and the 0 and 1 of arithmetic. In early times this was thought to have mystical significance.

The formula for e^{ix} enables us to calculate all the roots of any non-zero complex number. The roots of the number 1 are especially important in connection with a subject known as "digital signal processing." For each natural number n, the number 1 has n distinct n^{th} roots, and these constitute a group under the operation of multiplication (Chapter 12). There are certain of these numbers with the property that their powers give every member of the group. Such elements are called generators of the group, and a group that has a generator is called a cyclic group. Any two cyclic groups of the same order (number of members) are essentially the same. Thus the roots of unity give us all finite cyclic groups.

These numbers have many properties that make them ideal for certain calculations. They are also interesting geometrically. When plotted as points in the complex plane they all lie on the circle of radius one, centered at the origin. If they are joined by lines, one gets a regular polygon of n sides. So the fifth roots of unity give us a regular pentagon (a five sided figure having all sides of equal length).

Chapter 18

Epilogue

At the time of this writing, 2012, there is no evidence that alien life, intelligent or other wise, actually exists. It is hard to believe that it isn't out there somewhere, but we have yet to find it. SETI researchers are listening, hoping to detect the radio, in some cases even visual, signals of an alien civilization. I don't think anyone expects a direct communication, but we might get lucky and overhear someone's internal messages; perhaps a spacecraft contacting its home planet or a radio broadcast that, like ours, leaks into space. Those with the right technology can tune in to our early radio and television shows. Perhaps our nearest neighbors are listening to the Lone Ranger on the radio, waiting impatiently to hear what the next episode will bring, and wondering what "Kemo Sabe" means.

There are active SETI programs in existence in a number of countries. Japan and Italy have active groups as does Canada and Russia, and in the United States the SETI Institute has an array in northern California listening for signals. This institute employs many researchers; among them is Jill Tarter who, many say, was the inspiration for the character Elinor Arroway, played by Jodie Foster in the movie *Contact*. Its array of forty-three radio telescopes, which they hope to increase in number, is named for Paul Allen, their principal benefactor. There is also a very active group at the University of California at Berkeley and at Harvard. The SETI League is a group of amateur radio astronomers that seeks to keep interest in SETI alive. Its president is Paul Shuch.

Should any of these groups hear something that is unmistakably a signal from an alien civilization the news will spread far and wide; the media will see to it. Undoubtedly there will be much debate about contacting the aliens, but since the distances and hence the times involved will be great, I don't believe much will happen immediately, but we will finally know that we are not alone. This fact will have a profound effect on human thinking.

There probably will be a call for us to get back into space. Some may even argue that we should have military bases on the Moon, just in case the aliens come here and try to start something. There are those who will play on our natural fear of the unknown to get control of large sums of government money, and spend it to set up weapon systems in space. On the positive side we might finally do what we should have been doing since the American Apollo program: learning to live and work on the Moon. I though we would have an observatory on the dark side (recall that the same side of the Moon is always facing away from the Earth) shortly after the Apollo program. It seemed like the natural next step. Instead we turned our attention to deeper matters like the power of pyramids to sharpen razor blades, the healing properties of crystals, and the social interactions of vampires, werewolves, and elves. Judging by currently popular television programs we are still concerned with these things.

The impact of an actual contact will be deeply felt, however, in connection with religion and philosophy. I imagine the Pope and other religious leaders will be asked to comment.

There are those who will say they always knew aliens existed because their sacred books told them so, and there are those who will deny the evidence no matter how strong and convincing it is.

Reactions to SETI projects can be amusing. On the 500th anniversary of Columbus's voyage to the new world NASA began a ten-year search for an intelligent signal. This had two parts, an all sky search and a search of some specific, nearby stars. Unfortunately Congress cut off funding for the project after only two years. One congressman stated that NASA "had failed to bag a single little green fellow."

Much earlier, in the 1970s, NASA had supported a small SETI program. In 1978, Senator Proxmire of Wisconsin gave the program a Golden Fleece award. This was his way of saying it was a waste of government funds. SETI scientists responded by nominating him for membership in the Flat Earth Society. After a lengthy meeting with Carl Sagan, the Senator gave the project his support.

Will the human race ever have face-to-face contact with an alien race? Who can say? Is SETI a wasted effort because our technology is too crude? This is a question that experimentalists face all the time. Galileo tried to measure the velocity of light using the technology of his time: covered lanterns on adjacent hills as his light source and his pulse as his clock (Kaufmann 1994: 80). Today, with 20-20 hindsight, we can see that the experiment was doomed from the start. But Galileo couldn't know that and I think he deserves considerable credit for having the imagination and the courage to try; courage because there are always those ready and eager to criticize anything they themselves don't think of. To paraphrase Shaw: Those who can, do; those who can't, criticize.

The SETI people may be in a similar situation. The technology of our alien neighbors may be so far beyond us that we can't even imagine a viable means of detecting them. Our current means of searching for signals may be woefully inadequate, but we use the tools we have to try to solve the problem because the answer matters to so many of us.

Again the searchers deserve considerable credit for having the imagination and the courage to try.

The problem of communicating with an alien race is, as indicated in some of the chapters above, a complex and challenging one. Even the role of mathematics is uncertain. The world of mathematics is not the real world. It is a world of abstractions and idealizations. To those who doubt this I need only recall (Chapter 11) the theorem of Banach, Tarski, and Hausdorf: *Given a sphere the size of a pea, we can slice it into a finite number of pieces, reassemble the pieces, and obtain a sphere the size of the sun.*

This never happens in the real world. I might mention that the spaces where paradoxical decompositions of the sphere exist are those where the group (see the Remark in Chapter 12) of motions is of a certain type (Bruckner and Ceder 1975).

The thing that makes mathematics different is that it has the habit of becoming useful either in solving real-world problems or in modeling some aspect of the real world. But the abstractions and idealizations found in mathematics were made by human beings. Our mathematics is as much a part of our humanity as is our music and our art. If we contact a race that can count, then we can communicate to its members a great deal of our mathematics, and in doing so we say a great deal about ourselves.

Our geometry may show them that we are visually oriented; our sense of sight dominates. It may also give them a good idea of how the world looks to us. Our calculus may show them that at the human level the world is “continuous,” and that we have a good grasp of the properties of motion at this level. They might also deduce that we know something of the science of mechanics. Our knowledge of the foundations of differential and integral calculus implies that we have a deep understanding of the nature of infinite processes. This is not a trivial accomplishment. What they will deduce about us from our set theory is a little harder to say. Perhaps our preoccupation with infinite sets somehow reflects our awareness of the finiteness of our lives. Our desire to know the infinite seems somewhat like a religious impulse, and this branch of mathematics seems a little like theology.

As we have seen (Chapter 12) progress in algebra required a higher level of abstraction than previously needed. Group theory and the other structures found in modern algebra require a more sophisticated view of mathematics.

Whether or not we will share these things with an alien race is questionable. Groups, rings, fields, etc. are defined by sets of axioms. We were led to these axioms over time by calculations and experience. It is not clear that an alien race will extract these same axioms from their experiences.

There are many different areas of topology. Perhaps the best known is network topology, now known as graph theory, which arose from Euler's solution to the Konigsberg bridge problem: Konigsberg was a city situated on both banks of a river in which there were two islands. These were linked to each other and to the shores of the river by seven bridges. Many wondered if it were possible to find a closed path that crossed each bridge exactly once. Euler, an eighteenth-century Swiss mathematician that some have called "the master of us all" (Dunham 1999), showed that this was impossible.

Another problem in this area that was once famous is this: Consider a map of the forty-eight contiguous states. Suppose you want to color each state so that the boundaries between them are clear. To do that, two states that share a border, like California and Arizona, must be different colors. With that understanding, what is the minimum number of colors you would need?

More generally, given any map, not necessarily of a real country, and as complicated as you wish, what is the minimum number of colors you'd need to color the map subject to the condition stated? The answer is four, and it took about a century before this was proved.

A more sophisticated area is algebraic topology. Its origins are complicated. In complex analysis, the study of functions defied on the complex numbers, certain formulas could be interpreted as functions if looked at as being defined on a surface in a higher dimensional space. This approach proved illuminating and fruitful and a great deal of effort went into the study of these rather complicated surfaces; they are called Riemann surfaces after the German mathematician who first introduced them.

Another problem that attracted the attention of mathematicians to this area was that of trying to understand a curious observation of Descartes made in 1639. A cube, for example, has 6 sides or faces, 12 edges, and 8 vertices. Calling these F, E, and V respectively, we see that $F - E + V = 2$. This simple formula seems to hold for any polyhedron, and people wanted a proof. Investigating this, and similar, problems led to a vast field of mathematics. Here again, the theory of groups proves useful.

We have already mentioned general topology (Chapter 16). This involves a careful study of the properties of space. One of its goals was to try to find a reasonable way to assign a "dimension" to various sets.

All of these areas demonstrate our strong affinity for visual problems and the different aspects of geometry. There is no doubt that this interest has led to a great deal of valuable mathematics. And maybe this reliance on the visual is something we will share with any alien race that is interested enough in the cosmos to develop the radio telescope. A race that relies more heavily on the chemical senses, as many animals seem to do, may be more interested in chemistry and may develop very different mathematics. Modern chemists make use of quantum theory and the associated mathematics. I had lunch with a colleague from the chemistry department once who told me he saw no reason for his students to learn calculus; a standard course for freshmen. He wanted them to know linear algebra and the associated matrix theory.

Linear algebra is concerned with systems of linear equations and the Euclidean spaces that arise in connection with these systems (Chapter 14). Certain functions from a space of dimension n, to a space of dimension m, can be represented by a rectangular array of numbers having n columns and m rows called an m x n (read "m by n") matrix. There is a lot of mathematics here and a race that favors the chemical senses may develop their mathematics along these lines perhaps never thinking of calculus, topology, or functional analysis. Here again, however, the complex numbers play an important role.

We have discussed how the relatively short day-night cycle may have led to the idea of ordinal numbers and the process of counting (Chapter 3). Attempts at recording the longer seasonal cycle led to the calendar and problems of modular arithmetic (Chapter 3). A deeper perception of time seems to be related to consciousness. Can there be intelligent beings that are not conscious? Such beings, if they exist, might be aware of cycles and yet not really aware of time. Differential calculus and Newtonian physics might never occur to them. The psychologist Julian Jaynes has suggested that some ancient peoples were, in fact, not conscious (Jaynes 2000).

We would like, of course, to give a more complete picture of humanity to those we contact. It is at this point that the problem of communication becomes much more complicated. It seems likely that basic mathematics and physical science would be understood by any race that has the equivalent of the radio telescope. Thus it seems likely that precise scientific information can be exchanged; the ability to do this is what Oehrle and I tried to accomplish in constructing our language. How far we can go beyond that, however, is very uncertain. It is here that, I think, we need the input of researchers in the social and humanistic scientists.

REMARK: Ramanujan

There are people with incredible numerical abilities. A pair of mildly mentally challenged twins often amused themselves by reciting six-digit primes to one another (Sacks 1998). I dare say that the average mathematician would be hard pressed to come up with even one six digit prime.

The Indian genius Ramanujan, whose talent was first recognized by G. H. Hardy, was remarkable in his numeric ability. He was also a creative mathematician.

Hardy tells this story about him:

> I remember once going to see him when he was lying ill at Putney. I had ridden in taxi-cab number 1729, and remarked that the number seemed to me rather a dull one, and that I hoped it was not an unfavorable omen. "No," he replied, "it is a very interesting number; it is the smallest number expressible as a sum of two cubes in two different ways."

This means that there are a pair of whole numbers, a and b, such that 1,729 is equal to $a^3 + b^3$, and that there is a different pair, say c and d, such that 1,729 is equal to $c^3 + d^3$, and 1,729 is the smallest number for which this is so.

Where does this remarkable ability come from? Could there be an alien race in which every individual has this talent? I suppose that's like asking if there is an alien race in which all members are musical prodigies, which is possible but highly unlikely.

Appendix I

Infinite Sets

In the nineteenth century mathematicians regarded many mathematical objects as potentially infinite. The set of natural numbers, for example, was regarded in this way. No matter how many of these numbers you wrote down, there were always more. It was Gerog Cantor (1845–1918) who first made a systematic study of sets that were actually infinite. This was quite a controversial idea in its time, and Cantor received a great deal of nasty criticism. Infinite sets, sets that are actually infinite and not just potentially infinite, now play a central role in modern mathematics.

Cantor's discovery that infinity is not a simple on/off kind of property came about when he tried to set up a one-to-one correspondence between the natural numbers and the real numbers. In order to show that this cannot be done it suffices to show that there is no one-to-one correspondence between the natural numbers and the real numbers that are between zero and one, i.e., those that are strictly larger than zero and strictly less than one. Suppose that we had such a correspondence, let's call it F, between these two sets. Then we could make a list:

$F(1) = 0.a_{11}a_{12}a_{13}a_{14}a_{15}a_{16} \ldots$
$F(2) = 0.a_{21}a_{22}a_{23}a_{24}a_{25}a_{26} \ldots$
$F(3) = 0.a_{31}a_{32}a_{33}a_{43}a_{35}a_{36} \ldots$
And so on.

Since we are assuming that F is a one-to-one correspondence, every real number that is between zero and one must appear somewhere on this list. We shall deduce a contradiction by constructing a real number between zero and one that is not on this list.

Let's call the number $b = 0.b_1b_2b_3b_4b_5b_6 \ldots$. We choose b_1 to be 1 unless a_{11} is 1; if that is the case, we take b_1 to be 2. Next we choose b_2 to be 1 unless a_{22} is 1; if that is the case, we take b_2 to be 2. Next we choose b_3 to be 1 unless a_{33} is 1; if that is the case, we take b_3 to be 2. Continue in this way. The number b constructed in this way is certainly between zero and one. This number is not the same as F(1) because b_1 is not equal to a_{11}. It is not the same as F(2) because b_2 is not equal to a_{22}, and so on.

Clearly b is not the same as any of the numbers on the table and so we have reached a contradiction. It follows that no one-to-one correspondence between the natural numbers and the real numbers can exist.

A set that can be placed in one-to-one correspondence with the natural numbers is called a denumerably infinite set. A set that is either finite or denumerably infinite is called a countable set. We have just shown that there are sets that are uncountable.

Given any set, say S, we will show that there is no one-to-one correspondence between S and its power set, P(S), the set of all subsets of S. The terminology comes from the fact that a finite set containing, say, n elements, has 2^n subsets (Chapter 12). So if S is a denumerably infinite set, then P(S) must be uncountable; there is, in fact, a one-to-one correspondence between the real numbers and the power set of the natural numbers.

To prove our claim we first suppose that for some set S we have a one-to-one correspondence, call it g, from S to P(S). Then for each s in S, g(s) is in P(S); i.e., g(s) is a subset of S. Let's call g(s) the image of s. The element s might belong to its image or it might not. Let S_0 consist of all elements of S that do not belong to their images (compare to Russell's paradox discussed in Chapter 12). Since S_0 is a subset of S it belongs to P(S). Since g is a one-to-one correspondence, there is a element s_0 in S such that $g(s_0) = S_0$.

Now we ask the question, "Does s_0 belong to its image?" If you say "no", then s_0 is in S_0 by definition of S_0. But S_0 is the image of s_0. So if s_0 doesn't belong to its image, then it does.

On the other hand if we say that s_0 is in its image, then s_0 belongs to S_0, and, from the definition of the set S_0, s_0 does not belong to its image.

The only conclusion we can draw is that the one-to-one correspondence g between S and P(S) does not exist.

We have said that a set is infinite if, and only if, it can be placed in one-to-one correspondence with one of its proper subsets. We all know that no finite set has this strange property, so if a set does have it, then the set must be infinite. Now suppose we are given an infinite set. How do we know we can find the kind of correspondence we said we could? There are two cases.

First, suppose that we have a denumerably infinite set A. Then we have a one-to-one correspondence, f, say, from the natural numbers to A. So f(1) is a point of A that we might call a_1, and f(2) is another point of A that we'll call a_2, and f(3) is yet another point of A, call it a_3, etc. In other words we have given each member of A a name, $a_1, a_2, a_3, \ldots$, and $A = \{a_1, a_2, a_3, a_4, \ldots\}$.

Clearly $\{a_2, a_3, a_4, a_5, \ldots\}$ is a proper subset of A. If we define $g(a_n)$ to be a_{n+1}, so $g(a_1) = a_2$ and $g(a_2) = a_3$, and so on, the g is a one-to-one correspondence between A and its proper subset $\{a_2, a_3, a_4, \ldots\}$.

The second case, when A is an uncountable set, is a little more complicated. Suppose A has a denumerably infinite subset $\{a_1, a_2, a_3, \ldots\}$. We may define g as follows: For each a_n set $g(a_n)$ equal to a_{n+1}. For each point b of A not in the denumerable set, set $g(b) = b$. Then g is a one-to-one correspondence between A and the set A with the element a_1 removed.

Let us now show that any infinite set A has a denumerable subset. Choose an arbitrary element of A and call it a_1. The set A with a_1 removed is still infinite, so it contains an element we'll call a_2. Now A with the two elements a_1 and a_2 removed is still infinite, so we can choose a point here we'll call a_3. Continue in this way to get a subset of A that is in one-to-one correspondence with the natural numbers; i.e., we get a subset of A that is denumerably infinite.

There is a possible objection to the last paragraph of our proof. We haven't really specified which element of A we are to pick at each stage of our construction; we just said choose some element of the set. So we really didn't define the subset of A that we are talking about. There is an axiom of set theory that enables us to avoid this problem. This is called the axiom of choice.

It states that for any non-empty family of non-empty sets there is a function that assigns to each set in the family a member of that set; so it chooses from each set a member of that set. This axiom, once highly controversial, is now known to be consistent with, and independent of, the other axioms of set theory. A great many fundamental results of analysis, algebra, and topology depend on this axiom, and many of these results are actually equivalent to the axiom.

Given our infinite set A, let f be a function that assigns to each non-empty subset of A a member of that set. Then f(A) is a well-defined element a_1 of A, and A with a_1 removed is a non-empty subset of A. Thus f(A with a_1 removed) is also a well-defined element of A. Continuing in this way we get a denumerable subset of A.

The set of real numbers is a union of two sets: The rational numbers (all whole numbers and quotients of whole numbers), and the irrational numbers. These two sets have no common elements; i.e., there is no number that is both rational and irrational.

It can be shown that the set of rational numbers is countable, and it can be shown that the union of two countable sets is countable. We have seen that the set of real numbers is not countable. It follows that the set of irrational numbers is uncountable. The irrational numbers are far more numerous than the rational numbers. A poetic description of this situation was given by E. T. Bell: "The rational numbers are spotted along the real line like stars against a black sky, and the dense blackness of the background is the firmament of the irrationals" (Simmons 1963: 37).

Appendix II

Mars

The most influential proponent of an advanced Martian civilization was Percival Lowell. In 1893 he built an observatory in Flagstaff, Arizona on what has come to be called Mars Hill. Although many astronomers dismissed his claims as nonsense, his lectures were packed and his books, *Mars* (1895), *Mars and Its Canals* (1906), and *Mars as the Abode of Life* (1908), became best sellers. He did have some support in the astronomical community, and when he traveled in Europe he often stayed at the home of a well-known astronomer whom he referred to as "my Martian friend" (Harrison 1997: 230). Schiaparelli (Chapter 1) eventually became convinced that the canals he "discovered" were waterways, although he did not go so far as to say that they were artificial constructions.

In August 1924, during a period when the Earth and Mars were as close as they ever get (the distance between the two planets varies) civilian and military broadcasters proposed a three-day radio silence to allow us to listen to any intelligent radio signal from Mars, and the U.S. Army designated its chief signal officer as the one who would try to decode any Martian message. Wireless operators in both Britain and Canada reported hearing several unexplained beeps. In the Swiss Alps a light ray was reflected off the snow covering Jungfrau and directed at Mars as a kind of greeting (Sobel 2005: 130).

Belief in a Martian civilization slowly faded, but in 1928 several prominent astronomers, including one from

Harvard and one from Princeton, endorsed the likelihood of Martian vegetation and the possibility of some form of animal life (Grinspoon 2003: 43).

H. G. Wells published his novel *The War of the Worlds* in 1898, and, forty years later, Orsen Welles broadcast a version of this classic on *The Mercury Theater of the Air.* Although the program began with a disclaimer asserting that this was fiction, many listeners became convinced that they were hearing breaking news. There was widespread panic. Some fled their homes while others barricaded themselves inside. This was a time of international unease due to the rise of Nazi aggression in Europe and this may have had some effect on people's reactions. The idea of foreign invasion resonated with many at this time in history.

Still the program itself did inspire fear. It was aired in Ecuador in 1949, causing panic there. When people learned it was fiction the reaction was violent. The radio station was torched. Another rebroadcast, in 1988, frightened listeners in northern Portugal. These incidents are discussed in Harrison 1997 (231–33). Many people claim that this is why the governments of the world suppress information about visiting UFOs: they fear widespread panic.

In 1877, the same year Schiaparelli claimed he had seen "canali," the American astronomer Asap Hall discovered the two moons of Mars. The chariot of the Roman god of war was drawn by two horses named Phobos (Fear) and Deimos (Panic), and Hall gave these names to the two moons. It was found by Bevan Sharpless in the 1940s that the orbit of Phobos, the larger of the two moons, was decaying. The rate of decay, however, was difficult to explain. After considering and rejecting various possible causes, the Russian astronomer Shklovsky concluded that the moon was much less dense than most assumed. To account for this low density he made the radical suggestion that the moon was hollow! If true, this would mean that the moon was an artificial satellite.

In their book Sagan and Shklovsky say that it might be the remnant of an ancient Martian civilization (Sagan and Shklovskii 1966: ch. 26). More radical ideas were put forth by various people. Hall discovered the moons in 1877 using a 26-inch telescope. Fifteen years earlier, when viewing conditions were much better, the Danish astronomer d'Arrest made a careful search for Martian moons using a larger telescope. How could he have missed them? Perhaps, it was said, because they weren't there. Maybe they were launched sometime between 1862 and 1877. They weren't the remnants of an ancient civilization but the products of a living, active alien race (Webb 2002: 39; Sagan and Shklovskii 1966: 374).

These wild speculations were discredited when spacecraft flybys showed that Phobos is just a large rock, probably a captured asteroid. But belief in a Martian civilization was not so easily stifled. Some saw a giant face in the photos sent back, and others saw nearby pyramids. Later photos showed that these things were just effects of the sunlight illuminating natural geologic features (Webb 2002: 40–41).

In 1996 a meteorite from Mars, ALH84001, was examined by scientists who claimed it contained remnants of bacterial microfossils. Thus, life may have began on Mars and been carried here in this way. Needless to say, this claim has been met with a great deal of skepticism (Webb 2002: 45).

Appendix III

The DeVito–Oehrle Language

Here I will give a brief overview of the original paper that I wrote with my colleague Richard Oehrle. It contains the details of how we might construct a language based on some mathematics, and some elementary physics and chemistry. I have emphasized the scientific parts of the paper because they contain the most interesting ideas.

The problem of how to communicate with the members of an alien society has been discussed by many authors but only one, Hans Freudenthal (see Chapter 11), has constructed a language for this purpose. Freudenthal assumes nothing other than the ability to reason as humans do, and, because he assumes so little, it is necessary to communicate a great deal about the language itself before being able to communicate any interesting information. Here the problem is approached differently. Since it is likely that contact between our civilization and an alien one would be via radio, potential correspondents would have a basic knowledge of science. Such beings should therefore be able to learn a language based on fundamental science. It is assumed, more specifically, that our correspondents can count, understand chemical elements, are familiar with the melting and boiling behavior of a pure substance and understand the properties of the gaseous state. All of this should be known to any society capable of developing the radio telescope. By systematically using this common knowledge one can communicate notation for numbers and chemical elements and then communicate our basic physical units, i.e., the gram, the calorie, the degree (Kelvin), etc. Once this is done, more interesting information can be exchanged.

Introduction

The purpose of our paper was to show how a language can be developed, based on the rudiments of logic, mathematics, and physical sciences that would enable us to communicate with hypothetical intelligent aliens. There are, of course, other reasons for constructing such a language. First, in the not very distant future, we shall want to involve the computer in scientific problems at a level higher than that now possible. This will require a language that incorporates science into its basic structure. Natural languages, i.e., those spoken by human beings, are too complicated for this and are unsuitable in other ways as well. Secondly, it is necessary to elucidate the manner in which real-world experience enters into language acquisition. This information could be valuable in the development of artificial intelligence, particularly in the area of machine translation. In developing a knowledge-based system to translate material in a certain area, it is important to know what minimal knowledge the system must have.

The idea of constructing a logic-based language goes back at least as far as Leibnitz (1646–1716). Late in the nineteenth century Peano (1858–1932) discussed such languages extensively, and his ideas strongly influenced Russell and Whitehead (Freudenthal 1960). These efforts had an important impact on the development of mathematical logic and on the foundations of mathematics (Freudenthal 1960). Computer scientists, in discussions of the problems encountered in the creation of programming languages, also drew on this work (Beckman 1981). However, the idea of using a logic-based language as a means of communication, the original intent of both Leibnitz and Peano, was forgotten until 1960. In that year, Freudenthal published a remarkable book called *Lincos* (a contraction of the words *lingua cosmica*) wherein he set himself the problem of designing a language suitable for communicating via radio with (hypothetical) intelligent extra-terrestrials. Since such beings would have no knowledge of our natural languages and could not be shown physical objects or demonstrations, there is nothing but logic on which to base a common language. Freudenthal showed that, if their thought processes are human-like, a language suitable for mutual communication could be taught to them.

Although Freudenthal had his critics, he also had supporters (Freudenthal 1974). His work attracted the attention of computer scientists (Beckman 1981) and those interested in SETI. A different situation occurs if it is assumed that extra-terrestrials have a technology, for this implies a detailed knowledge of the physical universe. We supposed that they have investigated certain basic scientific problems, e.g., the fundamental properties of matter and energy. How does one measure these things and how do they interact? An alien science may be radically different from our own but when intelligent races describe the same phenomena, particularly uncomplicated chemical or physical processes, the descriptions must be equivalent in the sense that they both really describe the same phenomenon. Hence, it does not seem unreasonable that these descriptions would be mutually understandable.

This additional assumption enables us to move rapidly to the point where interesting information can be exchanged. Communication of this kind must begin with modest goals. The best one can hope for is the facility to exchange precise information. It may be easy, but not very informative, to devise some scheme for saying that we live on a planet. To be able to state that the planet has a specified mass, radius, and atmospheric composition, etc. is to convey precise, useful information. This goal if different from Freudenthal's aim of constructing a complete language. The advantage of adopting a technical language is that exchanging valuable information can begin before the language is complete. Information is exchanged as the language is developed; the information received helps the user to carry the language further, whereas Freudenthal's Lincos is an actual language that contains words for many common concepts that must be learned as they are presented to be able to read further.

We have assumed that radio waves would be used and that the aliens have devices for sending and receiving such waves, while Freudenthal goes so far as to suggest that they might have sense organs for this purpose, at least for receiving. Many argue that the sense of sight must be universal and so television or picture messages should be sent. On the other hand, even given the sense of sight, there is no way of knowing how an alien would interpret a picture. There is no need to make an assumption one way or the other regarding the sense of sight though there is no doubt that its presence would simplify matters in many respects. For example, a picture of a balance scale with weights on it does not convey the gram to someone unaware of this unit; it merely shows, if it is understood, that the two weights are of the same or of different mass. So communication via television would change some of our problems and simplify others, but it would not eliminate them.

Format

The proposed technically based language may be presented as a series of stages rather than as dialogue, with the content of each stage dictated by prior stages. It turns out to be easy, once begun, to continue discussing mathematics or, again once begun, chemistry. The chief difficulty lies in going from a discussion of mathematics to a discussion of chemistry and from there to a discussion of physics, so we used certain scientific facts to provide "links" between these subjects. They are either distinctive phenomena, like the melting and boiling of a pure substance, or fundamental, like the notion of atomic weights, which enables us to do meaningful chemical calculations. Standard symbols for numbers, chemical elements, etc. have been used to enhance readability. Furthermore, any statement made in symbols is followed by a translation into English.

The attempt at communication may begin by assuming that the aliens are familiar with the process of counting. The rudiments of formal logic can then be developed. The basic symbols of this subject are the connectives "and," denoted by $\wedge$; "or," denoted by $\vee$; "not," denoted by ~; "implies" (the "if-then" of programming languages), denoted by $\rightarrow$; and "logical equivalence," denoted by $\leftrightarrow$. To illustrate the use of these symbols, suppose that n is a natural number (i.e., n = 1, 2, 3, 4, 5, . . .), then we may write: $(n < 2) \rightarrow (n = 1)$. In words: "If a natural number is less than 2, then it is the natural number 1." Another example $(n < 3) \rightarrow (n = 1) \vee (n = 2)$. In words: "If a natural number is less than 3, then it is either 1 or 2."

Greater flexibility is added to the language by introducing logical quantifiers: $\forall$ read "for all" and $\exists$ read "there is". Using these we may write, for example, $(\forall n)(n \leq n^2)$ and $(\exists n)(n=n^2)$. The first of these reads, "All natural numbers are less than or equal to their squares." The second reads, "There is a natural number which is equal to its square."

Next is the notation of set theory. Informally, a set is just a collection of any objects, e.g., the collection of all leaves on a certain tree is a set and, of course, this is not a leaf but a new object with properties all its own. Similarly, a flock of sheep is a set that has properties and characteristics of its own, different from those of an individual sheep. When possible, the objects forming a set are listed between two curly brackets. So the set consisting of the two letters a and b is written {a, b}, and the set of natural numbers is written {1, 2, 3, 4, . . .}, the latter set often denoted simply by N. The symbol ε is used to say that a particular object is in a set. So a ε {a, b}, b ε {a, b}; and 1ε N, 6 ε N, 23 ε N, etc. It is sometimes necessary to separate out from a given set those objects in the set that have a certain property. This gives us a new set. For example, the set of natural numbers greater than 5 many be written {n ε N | n > 5}. This is read "the set [that is the way curly brackets are read] of all natural number n such that [this is the meaning of the vertical bar] n > 5. One last example: When F is a flock of sheep and it is necessary to collect together into a new set those that weigh more than 30 pounds, we write {s ε F | s weighs more than 30 pounds}.

The Beginnings of a Language

Being in radio communication with an alien race alone implies that they have a sophisticated technology. The first assumption about their intellectual abilities will be that they can count and that the mental constructs that we call the natural numbers (i.e., 1, 2, 3, . . .) are understood. This collection has obvious properties that we may assume are also known. Specifically, the set is totally ordered (given any two distinct natural numbers, one is bigger than the other) and infinite. Counting is nothing more than setting up a one-to-one correspondence between a collection of objects, the things to be counted, and the first, say, n integers. Hence we may assume that the function concept is known, or at least one kind of function is known. Finally, it is supposed that the two shortcuts to counting—addition and multiplication—have been discovered. The idea of equality is implicit in this last assumption. For instance, one plus two is the "same thing" as three.

With this modest background we can begin developing our language. Our first step is to extend the natural numbers to the set Z (Chapter 4) of integers. Recall that Z contains all natural numbers, the number zero, and the negative of each natural number. We do this by working with equations whose solutions require subtraction. Next we introduce the set Q of rational numbers (Chapter 4) by discussing equations whose solution requires division.

We make immediate use of these constructions by discussing powers of ten. As I have already noted (Chapter 3) very large and very small numbers arise in science, and an alien society must have come to terms with this is some way. We concluded this section of the paper with a treatment of the function concept. Given two sets S and T, a function f from S to T is a "rule" that assigns to each s in S a unique element f(s) of T. This suffices for humans, but what is a "rule"? There is another way to get at this important concept. We first define the Cartesian product of the sets S and T : S x T = {(s, t) | s is in S, and t is in T}. A function f is a subset of S x T such that for every s in S there is a unique t in T for which (s, t) is in f. The set of all functions from S to T is denoted by Fun(S, T).

Matter

We now came to a crucial step in our language development. How do we move from the purely mental constructs of stages one and two and begin talking about objects in the real world. We begin by introducing a set At, first by simply listing it with known sets. So we write: N, Z, Q, At.

Next we name the elements of At by stating (x ε At) ↔ (x = 1 atom). The nature of the set At will probably not be clear at this point, but we persist. We say that there is a function Num with domain At and co-domain N; Num ε Fun(At, N). The "level" sets of Num are now given names:

H = {x in At | Num(x) =1}, He = {x in At | Num(x) =2}, Li = {x in At| Num(x) =3} and so on. We are, of course, discussing the chemical elements. In order to make this clear we want to present the periodic table, so important in chemistry (here, and perhaps, elsewhere). To begin we write Elm = {H, He, Li, Be, B, C, N, O, F, Ne, ..}, card (Elm) = 92. This, alone, might be suggestive.

Let T = {1, 2, 3, 4, 5, 6, 7} × {1, 2, 3, 4, 5, 6, . . . , 18} and now we state that there is a function Per from this "checkerboard" T into Elm ∨ {Ø}. More precisely, we state explicitly: Per ((1,1)) = H, read "H is in row one column one." Per ((2,1)) = Li, Per ((3,1)) = Na, Per ((4,1)) = K, and so on. We are giving the table column by column. So Per ((1,2)) = Ø (there is no element in row one column two) but Per ((2,2)) = Be, and Per ((3,2)) = Mg, Per ((4,2)) = Ca, and so on.

An alien "chemist" faced with a set of about 100 objects each assigned a natural number might, at least tentatively, assume that the chemical elements are being described; recall that these elements are universal (Chapter 13). If that is coupled with the tabular, periodic array presented next, it is very possible that an alien chemist would conclude that the elements are being discussed.

The specific meaning of these symbols can be reinforced by calling attention to the fact that some atoms combine. This requires a little more terminology. We say x = 1 atom C, if x is an atom and Num(x) = 6. We then introduce molecules: 1 atom O + 1 atom O → 1m, O; 1m O = O_2; 1 atom N + 3 atom H → 1m, NH_3 . The use of "→" in these equations is ambiguous because we have already used this symbol for "implies." We use it here because it is standard in chemistry and causes no trouble for humans. In an actual transmission we might want to devise some other notation.

We now want to communicate the meaning of our basic units of measurement; the gram, the degree (Kelvin), our units of volume, etc. This is why, besides the fact that we think that the chemical elements and the periodic table would be universally recognized, we began this message with a discussion of chemistry. Chemists everywhere are faced with the problem of relating their work in the laboratory, where large quantities of matter interact, with the changes taking place on the molecular level. Human chemists solved this problem by introducing a system of atomic weights and determining, experimentally, the Avogadro number. Alien chemists must also solve this problem; whether they have hands, claws, or tentacles, they cannot manipulate individual atoms. The problem is so critical that it seems reasonable that the aliens would be familiar with this number (expressed, of course, in their units). If this is so then a simple calculation based on the information given below will enable them to convert our gram into units which they understand.

We began by presenting our system of atomic weights. This is based on assigning an atomic weight of 12 to a certain isotope of carbon. For the sake of brevity we did not treat isotopes here, but our understanding of them is implied by the fact that many atoms have non-integer atomic weights. So amas ε Fun(At, Q), amas (1 atom, C) = 12, amas (1 atom, H) = 1.008, and so on. Now we extend this function to molecules: amas (1m, CO_2) = amas (1 atom, C) + amas (1m, O) = amas (1 atom, C) + 2 amas (1atom, O) = 44.

We cannot give a general definition of matter but we can introduce the family of all sets of atoms (Appendix I). We have denoted it by "Cmat," for "common matter," but this terminology merely serves as a mnemonic for terrestrial readers. Cmat = { L | L⊂At}, mass ε Fun(Cmat,Q) and finally [L ε Cmat] ∧ [L⊂C] ∧ [card (L) = 6.023×10^{23}] → mass(L) = 12 gram.

Many other examples should be given and are given in our paper.

Energy

The members of any technological society must be familiar with heat, with the fact that heat raises the temperature of a body and the fact that enough heat will change the state of a body. It is assumed that the aliens have devised a temperature scale, a method for measuring heat, and have studied the states of matter and the process of changing from one state to another. In particular, the gaseous state must be understood by any scientific society since failure to understand gases can lead to serious accidents. As far as we know, it was the study of gases that led humans to an understanding of absolute zero. It is assumed that the aliens share this knowledge with us. Our objective is to communicate our degree (Kelvin), our calorie, and the many related concepts mentioned above, beginning with the fact that chemical reactions often produce heat and sometimes require heat to get them started. For instance:

$$Fe + S + \Delta \rightarrow FeS, 12g, C + 32g, O \rightarrow 44g, CO_2 + \Delta$$
$$12g, C + 32g, O \rightarrow 44g, CO_2 + 94x10^3 \text{ cal}, \Delta$$

We have said that there is a quantity Δ that is produced by certain chemical reactions and that triggers some reactions. This Δ could not represent matter since all matter is accounted for in our equations. Perhaps this alone is enough to communicate the fact that Δ is our way of representing heat. If so, then our last equation already gives the alien race a means of finding our calorie. In any case we have tried below to give more clues to the meaning of Δ:

Temp ε Fun(Cmat, Q), (L ε Cmat) $\wedge$ (Temp(L) = 5 deg) $\longleftrightarrow$ (L@ 5 deg)
($\forall$L ε Cmat)(0 $\leq$ Temp(L)), ($\forall$L ε Cmat) (L + Δ) ε Cmat)
Temp (L) $\leq$ Temp (L + Δ)

We have said that we have a function Temp which is closely connected to this Δ introduced earlier and observed that Temp is non-negative and that adding Δ to matter increases Temp. Our degree can be found from the information given on the next line:

Ag @ 1232 deg = σ Ag, Ag @ 1233 deg = σ Ag
(1g, σ Ag @ 1234 deg) + 21.1 cal Δ → (1g, λ Ag @ 1234 deg)

The element silver is known. Our first two lines simply add the "adjective" σ to this symbol, and we state that Ag is σ Ag at various temperatures. However when a certain temperature is reached the addition of more heat to silver results in a change from σ Ag to λ Ag with no change in temperature. It would seem that anyone who has tentatively identified Δ with heat and Temp with temperature would conclude that σ Ag and λ Ag are two states of silver. The melting behavior of metals is so characteristic that this should be unmistakable. If so, then since the melting points of silver (in the paper we also gave these lines for the element lead whose melting point may also be known) is independently known to our aliens, and they know our gram, they can use this information to find our calorie and degree without using the information given earlier.
We have used metals because their melting points are not greatly affected by pressure. We now turn to a discussion of pressure.

Press ε Fun(Cmat, Q), (L ε Cmat) $\wedge$ (Press(L) = 1atm) $\longleftrightarrow$ (L @ 1atm)

This line just establishes our terminology. The notion of critical temperature can now be used to provide another means of finding our degree. This redundancy is intentional.

$(405 < t) \rightarrow (\forall\ p)(\ NH_3\ @\ t\ deg\ @\ p\ atm) = \gamma\ NH_3)$
$(t \leq 405) \rightarrow (\exists\ p)(NH_3\ @t\ deg\ @\ p\ atm) = \lambda\ NH_3)$

The first line says that when t is above 405 degrees, ammonia is a gas no matter what the pressure is. The second line says that when t is 405 degrees or less there is a pressure at which ammonia will liquefy.

We now use the concept of critical pressure to communicate the value of 1 atm.

$(112 < p) \rightarrow (NH_3\ @\ 405\ deg\ @\ p\ atm) = \lambda\ NH_3$
$(p < 112) \rightarrow (NH_3\ @\ 405\ deg\ @\ p\ atm) = \gamma\ NH_3$

The first line says that when the pressure is above 112 atm, ammonia at 405 degrees is a liquid. The second line says that when the pressure is below 112 atm, ammonia at 405 degrees will be a gas. From this information our unit of pressure can be found.

We now give some useful facts about water, the solvent that, as far as we know, is essential for life.

$(1g, \sigma\ H_2O\ @\ 273\ deg\ @\ 1\ atm) + 79.7\ cal\ \Delta \rightarrow (1g, \lambda\ H_2O\ @\ 273\ deg\ @\ 1\ atm)$
$(1g, \lambda\ H_2O\ @\ 373\ deg\ @\ 1\ atm) + 539\ cal\ \Delta \rightarrow (1g, \gamma\ H_2O\ @\ 373\ deg\ @\ 1\ atm)$

The first of these says that to change one gram of ice to one gram of water requires 79.7 calories, and the second says that to change on gram of water to one gram of steam requires 539 calories.

We have discussed two of the three variables necessary to describe the gaseous state. The third one is, of course, volume. It is not easy to give a general definition of volume. However, so many common phenomena, besides the study of gases (e.g., the expansion of liquids when heated, density, etc.), involve this concept that a society with a technology will have come to terms with this notion. So we take the position that the aliens understand volume and our task is to make clear that this familiar idea is what we are trying to communicate.

> L = 1g, λ H_2O, Vol(L @ 277 deg) = 1 cm^3, 1000 cm^3 = 1 liter
> Vol (4g, He @ 273 deg @ 1 atm) = 22.4 liter
> Vol (4g, He @ 273 deg @ 2 atm) = 11.2 liter
> Vol (4g, He @ 546 deg @ 1 atm) = 44.8 liter, and so on

We give the volume of 4 grams of helium (He) at 273 degrees and under 1atm, then show what happens when we double the pressure or double the temperature; we are stating Boyle's law and Charles's law.

Finally we shall give the ideal gas equation for the case of He and the value of the proportionality constant. If our units of pressure and temperature have been understood, the aliens can use the value of this constant to compute our unit of volume independently of our gram. Once that is done the gram can be recalculated from our definition of 1 cm^3. This gives another way of finding our gram:

> L = 4g, He, press(L) × Vol(L) = R × Temp(L), R = 0.8027 and so on

The final stage of the paper contains a technical discussion of the real numbers based on the work of Dedekind (Chapter 11). As we discussed in that chapter once we have communicated these numbers we can, in principle, communicate all of mathematical analysis.

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